Chapter 6
Work and Kinetic Energy

Up until now, we have assumed that the *force* is constant and thus, the *acceleration* is constant. Is there a simple technique for dealing with non-constant forces? Fortunately, the answer is, "Yes." In this chapter, we will introduce the following concepts:

1. work
2. kinetic energy
3. the connection between *work* and *kinetic energy*.

1 Work

The work performed by a force in the direction of a straight-line displacement is

$$W = Fs \qquad \text{(Constant force in the direction of straight-line displacement)}$$

where F is the force [N] and s is the displacement [m].

The SI units of *work* is $N \cdot m$ also called a *joule*.

$$1 \ N \cdot m = 1 \text{ J} \qquad 1\,\text{joule}$$

$$1 \text{ J} = 0.7376\,\text{ft} \cdot \text{lb}$$

$$W = Fs\cos\phi \qquad \text{(constant force, straight line displacement)}$$

$$W = \vec{F} \cdot \vec{s} \qquad \text{(constant force, straight-line displacement)}$$

$$W_{\text{tot}} = \left(\sum \vec{F}\right) \cdot \vec{s} = \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) \cdot \vec{s}$$

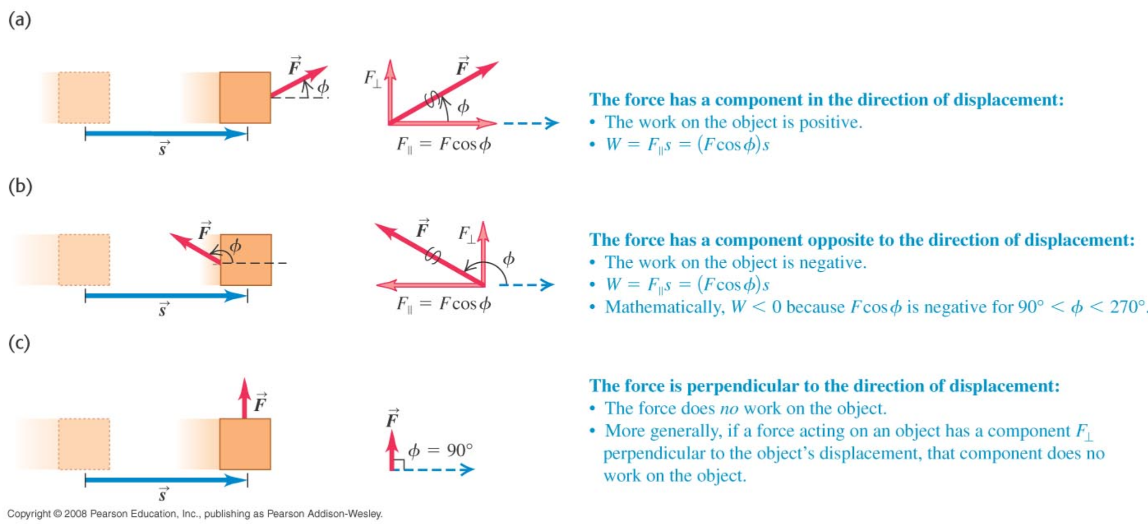


Figure 1: Figure 6.4 from University Physics

Ex. 3 A factory worker pushes a 30.0-kg crate a distance of 4.5 m along a level floor at constant velocity by pushing horizontally on it. The coefficient of kinetic friction between the crate and floor is 0.25. a) What magnitude of force must the worker apply? b) How much work is done on the crate by this force? c) How much work is done on the crate by friction? d) How much work is done by the normal force? By gravity? e) What is the total work done on the crate?

2 Work and Kinetic Energy

Constant force in the same direction as the displacement:

$$W = F \cdot s = ma \cdot s = m\left(\frac{v^2 - v_o^2}{2s}\right) s = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \tag{1}$$

where

$$K = \frac{1}{2}mv^2 \qquad \text{(K = the kinetic energy)}$$

So, the relationship between *work* and the *kinetic energy* is:

$$W_{\text{tot}} = K_{\text{f}} - K_{\text{i}} = \Delta K \qquad \text{(The Work-Energy Theorem)} \tag{2}$$

Ex. 24 You throw a 3.00-N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work-energy theorem to find a) its speed just as it left the ground and b) its maximum height.

3 Work and Energy with Varying Forces

Work resulting from the application of a varying force (i.e., a force that varies as a function of x), is the sum of the "incremental works" :

$$W = F_1\Delta x_1 + F_2\Delta x_2 + F_3\Delta x_3 + \cdots = \int_{x_o}^{x} F_x \, dx$$

For a constant force, $W = F_x(x - x_o)$.

Using *integral calculus*, we can calculate the work done by a *variable force* if we know the force as a function of x ($F_x(x)$).

$$W = \int_{x_o}^{x} F_x(x) \, dx$$

3.1 Hooke's Law and the work done by a spring

Hooke's Law is applicable to a wide number of physical systems. In many systems, the restoring force is proportional to the displacement from equilibrium. The tendency for a system to return to its "equilibrium" position" can be written as

$$F = -k\,x \qquad \text{Hooke's Law} \tag{3}$$

where x is the displacement from equilibrium ($x = 0$), and k is the spring constant.

N.B. In this section, the book correctly states that the *work done on the spring* is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. This is true if you use $F = kx$. However, in the application of Hooke's Law (Eq. 3), we are more interested in the *work done by the spring* on a system. This leads to the following equation:

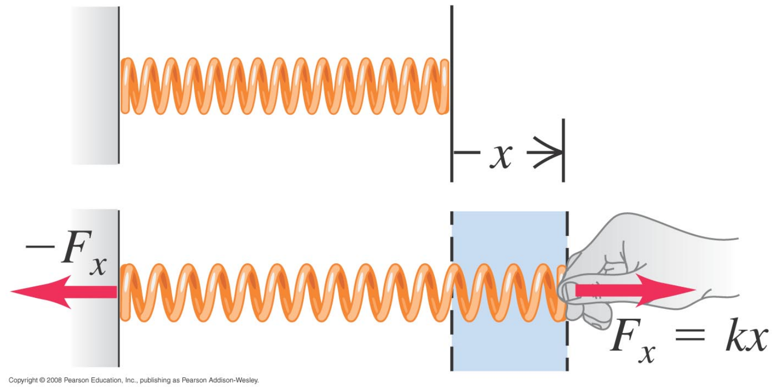


Figure 2: Figure 6.18 from University Physics showing the *work done on the spring.*

$$W = \int_{x_1}^{x_2} F_x \, dx = \int_{x_1}^{x_2} (-kx) \, dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 \qquad \text{Work done by the spring}$$

and this is consistent with Eq. 7.10 used in the next chapter.

Ex. 34 To stretch a spring 3.00 cm from its unstretched length, 12.0 J of work must be done. (a) What is the force constant of this spring? (b) What magnitude force is needed to stretch the spring 3.00 cm from its unstretched length? (c) How much work must be done to compress this spring 4.00 cm from its unstretched length, and what force is needed to compress it this distance?

Ex. 36 A child applies a force $\vec{F}$ parallel to the x-axis to a 10.0-kg sled moving on the frozen surface of a small pond. As the child controls the speed of the sled, the x-component of the force she applies varies with the x-coordinate of the sled as shown in Fig. E6.36. Calculate the work done by the force $\vec{F}$ when the sled moves a) from x=0 to x=8.0 m; b) from x=8.0 m to x=12.0 m; c) from x=0 to 12.0 m.

3.2 The work-energy theorem for a varying force

Finally, we take another look at the work-energy theorem for forces that vary with position.

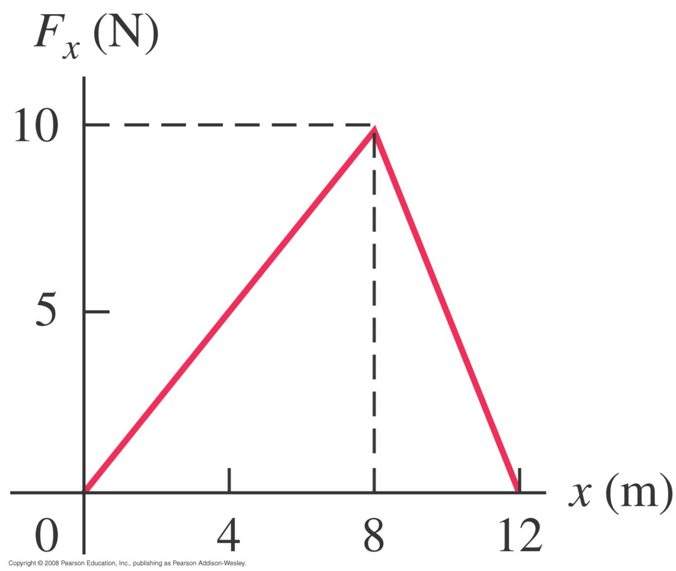


Figure 3: Figure E6.36 from University Physics 14$^{\text{th}}$ edition.

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2} m a_x\,dx = \int_{x_1}^{x_2} m\frac{dv}{dt}\,dx = \int_{x_1}^{x_2} m\frac{dv}{dx}\frac{dx}{dt}\,dx = \int_{x_1}^{x_2} m\frac{dv}{dx}v\,dx \quad (4)$$

$$W = \int_{v_1}^{v_2} mv\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad (5)$$

So, the *work-energy* theorem is the same as before even though the force is not constant. Revisit exercise 36 and calculate the final velocity of the sled, assuming it starts from rest.

4 Power

The definition of work makes no reference to the passage of time. We define a physical quantity called "power" that describes the *rate* at which work is done on the system. Similar to *work* and *energy*, power is also a scalar quantity.

When a quantity of work ΔW is done during a time interval Δt, the average work done per unit time or **average power** is defined to be

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} \qquad \text{(average power)}$$

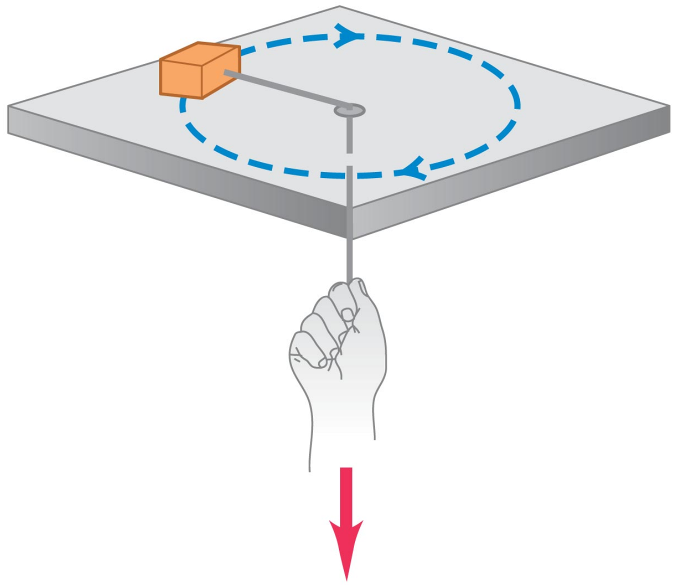
The rate at which work is done may not be constant, in which case, we can define the instantaneous power using the definition of the derivative:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad \text{(instantaneous power)}$$

Ex. 56 When its 75-kW (100-hp) engine is generating full power, a small single-engine airplane with mass 700 kg gains altitude at a rate of 2.5 m/s (150 m/min, or 500 ft/min). What fraction of the engine power is being used to make the airplane climb? (The remainder is used to overcome the effects of air resistance and of inefficiencies in the propeller and engine.)

Ex. 58 An elevator has mass 600 kg, not including passengers. The elevator is designed to ascend, at constant speed, a vertical distance of 20.0 m (five floors) in 16.0 s, and it is driven by a motor that can provide up to 40 hp to the elevator. What is the maximum number of passengers that can ride in the elevator? Assume that an average passenger has mass 65.0 kg.

Prob. 71 A small block with a mass of 0.0600 kg is attached to a cord passing through a hole in a frictionless, horizontal surface (Fig. P6.71). The block is originally revolving at a distance of 0.40 m from the hole with a speed of 0.70 m/s The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.10 m. At this new distance the speed of the block is observed to be 2.80 m/s. a) What is the tension in the cord in the original situation when the Block has speed $v = 0.70$ m/s? b) What is the tension in the cord in the final situation when the block has speed $v = 2.80$ m/s? c) How much work was done by the person who pulled on the cord?



Prob. 72 A proton with mass 1.67×10^{-27} kg is propelled at an initial speed of 3.00×10^5 m/s directly toward a uranium nucleus 5.00 m away. The proton is repelled by the uranium nucleus with a force of magnitude $F = \alpha/x^2$, where x is the separation between the two objects and $\alpha = 2.12 \times 10^{-26}$ N·m^2. Assume that the uranium nucleus remains at rest. a) What is the speed of the proton when it is 8.00×10^{-10} m from the uranium nucleus? b) As the proton approaches the uranium nucleus, the repulsive force slows down the proton until it comes momentarily to rest, after which the proton moves away from the uranium nucleus. How close to the uranium nucleus does the proton get? c) What is the speed of the proton when it is again 5.00 m away from the uranium nucleus?

N.B. Problem 72 can be done without integration once we introduce the concept of *conservative forces* and *potential energy functions* in chapter 7.