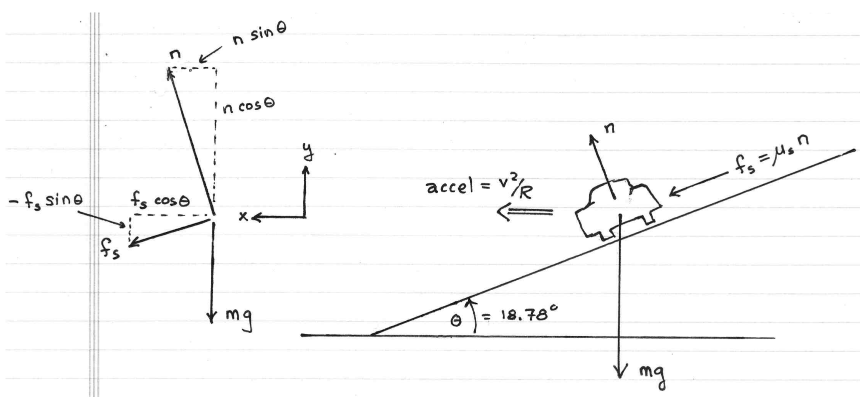
Chapter 5 Problem 99

A curve with a 120-m radius on a level road is banked at the correct angle for a speed of 20 m/s. If an automobile rounds this curve at 30 m/s, what is the minimum coefficient of static friction needed between tires and road to prevent skidding?

First of all, we can calculate the angle of the incline by solving $\sum F_x = mv^2/R$ and $\sum F_y = 0$ and finding that $\tan\theta = v_o^2/Rg$, where v_o is 20 m/s. Solving these equations simultaneously we find that $\theta = 18.78^o$. In this part of the problem where no *static friction* is required, the normal force is just $n = mg/\cos\theta$.

For the second part of the problem where the speed increases from 20 m/s to 30 m/s, a static friction force ($\mu_s n$) parallel to the banked road is now required. Let's draw a free-body diagram with a coordinate system and components of forces along the x and y directions.



Notice that the direction of the x axis is in the direction of the acceleration to help simplify the equations, $\sum F_x = mv^2/R$ and $\sum F_y = 0$.

$$\sum F_x = m\frac{v^2}{R} \quad \Rightarrow \quad f_s \cos\theta + n \sin\theta = m\frac{v^2}{R} \tag{1}$$

$$\sum F_y = 0 \quad \Rightarrow \quad -f_s \sin\theta + n \cos\theta - mg = 0 \tag{2}$$

Making the substitution for the *static friction*, $f_s = \mu_s n$, we can re-write equations 1 and 2 and factor out the normal force n in both equations and find:

$$n = \frac{mv^2}{R(\mu_s \cos\theta + \sin\theta)} \tag{3}$$

$$n = \frac{mg}{(-\mu_s \sin\theta + \cos\theta)} \tag{4}$$

Setting equations 3 and 4 equal to each other and defining $\beta = v^2/Rg$, we can solve for the coefficient of static friction μ_s:

$$\mu_s = \frac{\beta - \tan\theta}{1 + \beta\tan\theta} \tag{5}$$

If we make the following substitutions, $\beta = v^2/Rg$ and $\tan\theta = v_o^2/Rg$, Eq. 5 can be rewritten as:

$$\mu_s = \frac{v^2 - v_o^2}{Rg + \frac{v^2\,v_o^2}{Rg}} = \frac{30^2 - 20^2}{120(9.8) + \frac{(30^2)(20^2)}{120(9.8)}} = 0.337 \tag{6}$$

Notice that Eq. 6 gives us an equation for the minimum coefficient of static friction required to keep the car from slipping as a function of velocity v.

$$\mu_s = \frac{v^2 - v_o^2}{Rg + \frac{v^2\,v_o^2}{Rg}} \qquad \text{(minimum } \mu_s \text{ required)} \tag{7}$$

Let's plot this equation for a range of velocities between 10 and 35 m/s.

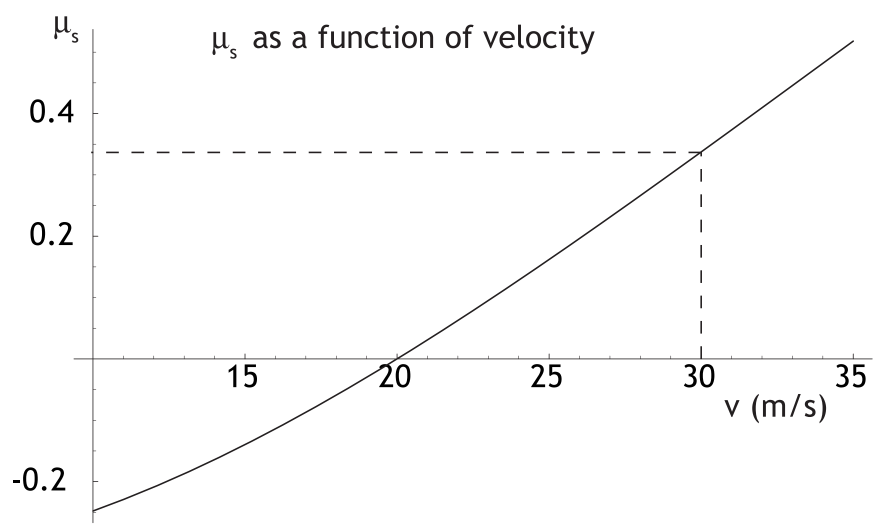


Figure 1: This figure shows the coefficient of static friction required to keep a car from slipping when the ramp is designed for a car traveling at 20 m/s ($\mu_s = 0$). *Equation 7 was plotted using Mathematica 5.1.*