Chapter 2
Motion Along a Straight Line

1 Displacement, time, and Average Velocity

This chapter, and the next one, deal with the branch of physics called kinematics. The physical quantities of interest (in one dimension) are:

- the initial position (x_1), and the final position (x_2),
- the initial time (t_1), and the final time (t_2),
- the initial velocity (v_1), and the final velocity (v_2), and
- the acceleration (a).

1.1 The displacement and time interval

With these physical quantities, we make the following definitions:

$$\text{the displacement} \qquad \Delta x = x_2 - x_1$$

$$\text{the time interval} \qquad \Delta t = t_2 - t_1$$

1.2 The average velocity

The average velocity for straight line motion is:

$$v_{\text{av}} = \frac{\Delta x}{\Delta t}$$

Note: Plot x as a function of t.
v_{av} only depends on the *initial* and *final* positions and times, and **not** on the motion in between the initial and final positions.

Figure 2.3

Exericse 4: Starting from a pillar, you run 200 m east (the $+x$ direction) at an average speed of 5.0 m/s, and then run 280 m west at an average speed of 4.0 m/s to a post. Calculate (a) your average speed from pillar to post and (b) your average velocity from pillar to post.

2 Instantaneous Velocity

The instantaneous velocity is the velocity v at a specific time t.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{(the instantaneous velocity)}$$

Note: Plot x as a function of t.
v describes the *instantaneous velocity* at only one point in space and time.

Figure 2.7

Figure 2.8

Exercise 7: A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by $x(t) = b\,t^2 - c\,t^3$, where $b = 2.40$ m/s^2 and $c = 0.120$ m/s^3. a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0$ s. b) Calculate the instantaneous velocity of the car at i) $t = 0$; ii) $t = 5.0$ s; iii) $t = 10.0$ s. c) How long after starting from rest is the car again at rest?

3 Average and Instantaneous Acceleration

The average acceleration a_{av} in one dimension is written as follows:

$$a_{\text{av}} = \frac{\Delta v}{\Delta t} \qquad \text{(the average acceleration)}$$

The instantaneous acceleration a in one dimension is the acceleration at a specific time t.

Figure 2.12

Figure 2.13

Exercise 16: An astronaut has left the International Space Station to test a new space scooter. Her partner measures the following velocity changes, each taking place in a 10-s interval. What are the magnitude, the algebraic sign, and the direction of the average acceleration in each interval? Assume that the positive direction is to the right. a) At the beginning of the interval the astronaut is moving toward the right along the x-axis at 15.0 m/s, and at the end of the interval she is moving toward the right at 5.0 m/s. b) At the beginning she is moving toward the left at 5.0 m/s, and at the end she is moving toward the left at 15.0 m/s. c) At the beginning she is moving toward the right at 15.0 m/s, and at the end she is moving toward the left at 15.0 m/s.

4 Motion with Constant Acceleration

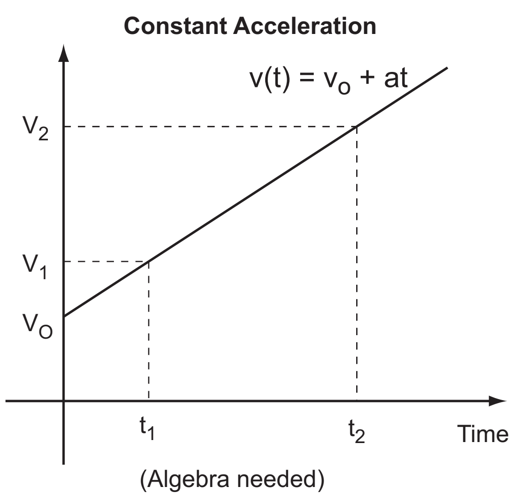
- It is now possible to describe the motion of an object traveling with a constant acceleration along a straight line.

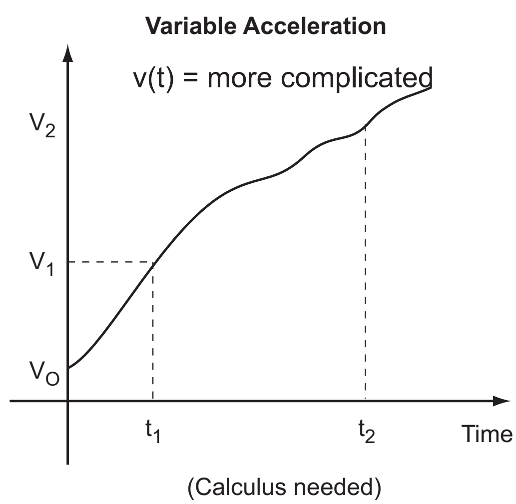
- There are 5 kinematical quantities to identify:

1.	x	the final position,
2.	v_o	the initial velocity,
3.	v	the final velocity,
4.	a	the constant acceleration, and
5.	t	the final time

- There are 4 kinematical equations to remember:
  1. $v = v_o + at$
  2. $x = \frac{1}{2}(v_o + v)t$
  3. $x = v_o t + \frac{1}{2}at^2$
  4. $v^2 = v_o^2 + 2ax$

These equations assume that $x_o = 0$ at $t_o = 0$.





- These equations assume that $x_o = 0$ at $t_o = 0$.
- Each of these four equations contains four of the five kinematical quantities.
- Once you've identified three kinematical quantities, you can solve for the fourth by using the equation that contains those four kinematical quantities.
- **In order to master these equations,** you must solve many problems.

Ex. 21: The fastest measured pitched baseball left the pitcher's hand at a speed of 45.0 m/s. If the pitcher was in contact with the ball over a distance of 1.50 m and produced constant acceleration, (a) what acceleration did he give the ball, and (b) how much time did it take him to pitch it?

Ex. 31: The graph in Fig. E2.31 shows the velocity of a motorcycle police officer plotted as a function of time. a) Find the instantaneous acceleration at $t = 3$ s, at $t = 7$ s, and at $t = 11$ s. b) How far does the officer go in the first 5 s? The first 9 s? The first 13 s?

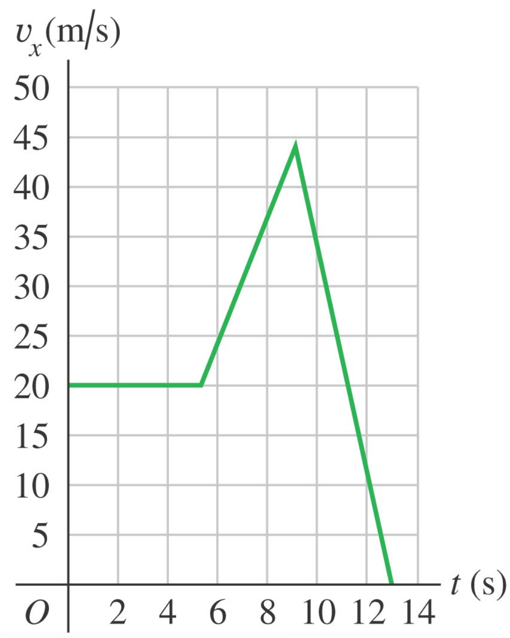


Figure 1: This is Fig. E2.31 showing the velocity profile for Exercise 31.

5 Freely Falling Bodies

- When discussing freely falling bodies, the motion of the object is confined to the y direction.
- The four equations (for constant acceleration) that were used in the x direction can now be applied to motion in the y direction. As a matter of convention, we will choose $+y$ to be in the upward direction.
- That means that the acceleration a will be written as $a = -g$, where $g = 9.8$ m/s^2. So, in the four kinematical equations up above (see section 4), you can replace all the *a's* with *-g's*. For example, the first equation becomes $v = v_o - gt$ in the y direction, and so on for the other equations.
- There are many examples in this section, and I encourage you to work them out. Everything you learned about using the 4 kinematical equations in the x direction is equally applicable in the y direction. The only difference is that for freely falling bodies near the surface of the earth, you can set the acceleration in the y direction to $a = -g$.
- There are 4 kinematical equations to remember for freely falling bodies:
  1. $v_y = v_{oy} - gt$
  2. $y = \frac{1}{2}(v_{oy} + v_y)t$
  3. $y = v_{0y}t - \frac{1}{2}gt^2$
  4. $v_y^2 = v_{oy}^2 - 2gy$

Ex. 40: **Touchdown on the Moon.** A lunar lander is making its descent to Moon Base I (Fig. E2.40). The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0 m above the surface and has a downward speed of 0.8 m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches the surface? The acceleration due to gravity on the moon is 1.6 m/s^2.

Pr. 63: A ball starts from rest and rolls down a hill with uniform acceleration, traveling 200 m during the second 5.0 s of its motion. How far did it roll during the first 5.0 s of motion?

Pr. 82: A ball is thrown straight up from the ground with speed v_o. At the same instant, a second ball is dropped from rest from a height H, directly above the point where the first ball was thrown upward. There is no air resistance. a) Find the time at which the two balls collide. b) Find the value of H in terms of v_o and g so that at the instant when the balls collide, the first ball is at the highest point of its motion.

6 Velocity and Position by Integration

This section is optional, and is meant for those who have already learned some integral calculus. Here is a brief discussion in case you know some integral calculus. We have already seen that the instantaneous velocity can be written as $v_x = dx/dt$, and that the instantaneous acceleration can be written as $a_x = dv_x/dt$.

Let's assume that we know the instantaneous acceleration along the x direction, a_x. Then, working backwards, we can find the change in velocity by using the definition of the instantaneous acceleration $a_x = dv_x/dt$.

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x \, dt \tag{1}$$

Likewise, we can now find the the change in position along the x direction by using the definition of the instantaneous velocity $v_x = dx/dt$.

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x \, dt \tag{2}$$

In both of these integrals, if we let t_1 and t_2 represent the initial and final times respectively, (e.g., 0 and t respectively), then $v_{2x} \to v_x(t)$ and $v_{1x} \to v_{ox}$. Likewise, in the displacement equation, $x_2 \to x(t)$ and $x_1 \to x_o$.

Then, in general, the above two integrals become:

$$v_x(t) - v_o = \int_0^t a_x(t)\,dt \tag{3}$$

$$x(t) - x_o = \int_0^t v_x(t)\,dt \tag{4}$$

These two integrals are the most general expression by which the equation of motion $x(t)$ can be calculated once you know $a_x(t)$. Where does $a_x(t)$ come from? Either from direct measurement of the acceleration as a function of t, or from Newton's 2^{nd} law, $\sum F_x = ma_x$.

6.1 Constant Acceleration

If the acceleration is a constant, we see that Eq. 3 reduces to

$$v_x(t) = v_o + at \tag{5}$$

Substituting Eq. 5 into Eq. 4 we find that the displacement $x(t)$ becomes:

$$x = x_o + v_o t + \frac{1}{2}at^2 \tag{6}$$

So, Eqs. 3 & 4 represent the most general approach to finding the velocity and position as a function of time <u>once</u> you know the acceleration, and $a_x(t)$ does not have to be constant.