Chapter 1
Units, Physical Quantities, and Vectors

1 The Nature of Physics

- Physics is an experimental science.
- Physicists make observations of physical phenomena.
- Physicists try to find patterns and principles that relate to these phenomena.
- These patterns are called physical theories. And, over time, if they become well established, they are called physical laws or principles.
- The development of physical theories is a two-way process that starts and ends with observations or experiments.
- Physics is the *process* by which we arrive at general principles that describe how the physical universe behaves.
- No theory is every regarded as the final or ultimate truth. New observations may require that a theory be revised or discarded.
- ***It is in the nature of physical theory*** that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.
- Theories typically have a ***range of validity***.

1.1 From the Six Ideas that Shaped Physics

The Nature of Science

1. It is part of human nature that we strive to discern order in the cosmos and love to tell each other stories that *explain* what we discern, using ideas common to our experience.
2. Scientist express their stories in the form of **conceptual models**. A good physical model captures a phenomenon's *essence* while remaining small and

simple enough to grasp. Model building is an essential aspect of science.

3. Inventing a model is less an act of discovery than an act of imagination.
4. **Theories** are the grand models that are able to embrace a huge range of phenomena.
5. **Science** is, more than anything, a uniquely powerful process for generating and evaluating models.
6. A discipline becomes a science when the following four elements come together:
   - A sufficiently large community of scholars.
   - A commitment to logical consistency as an essential feature of all models.
   - An agreement to use **reproducible experiments** to test models.
   - An overarching theory rich enough to provide a solid context for research.
7. A scientific theory needs to be falsifiable.
8. When a theory is extended beyond its ability to explain the phenomena at hand, it usually gets replaced by a more powerful model or theory. Such was the case with Newton's Laws, and the refinements that came with the Theory of Relativity.

1.2 The Standard Model

Protons are not point-like particle. They are made up of constituent particles called quarks. The quarks, leptons, and the force carriers for three of the forces form a simple periodic table called the **Standard Model**.

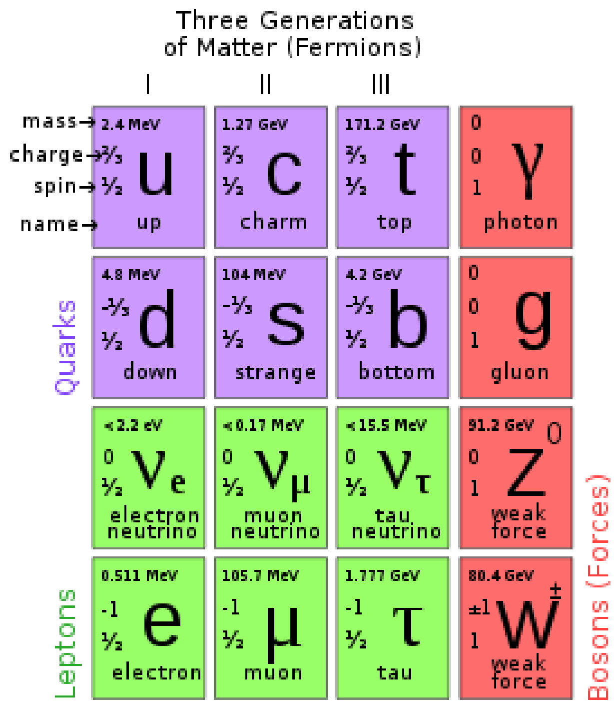

Figure 1: The Standard Model showing the three families of quarks and leptons, along with the force carriers for three of the four forces–electromagnetism, weak and strong forces.

Exercise: The proton is constructed of two u quarks and a d quark. The neutron is constructed of two d quarks and a u quark. The Δ^{++} "particle" is an unstable quark triplet similar in mass to the proton but with double the charge. What quarks must it contain?

2 Solving Physics problems

- "You don't *know* physics unless you can *do* physics."
- Some guidelines
  1. Identify the relevant concepts
  2. Set up the problem
  3. Execute the solution (i.e., do the math)
  4. Evaluate your answer (i.e., Does it make sense? Can you do a consistency check?)

3 Standards and Units

- A **physical quantity** is a number used to describe a physical phenomenon (usually a measurement).
- A physical quantity has units (e.g., mass, length, or time).
- In science, we use the SI (Système International) set of units
  1. Mass – the kilogram
  2. Length – the meter
  3. Time – the second

The kilogram, meter, and second are base units in the SI units. Combinations of these units can be used to describe other physical quantities such as velocity *(m/s)*, and acceleration *(*m/s^2). Sometimes the string of units gets to be so long that we contract them into a new unit called a *derived* unit. For example,

$$\text{A unit of force has base units of} \quad kg \cdot \frac{m}{s^2} \quad \rightarrow \quad \text{newton} \quad \text{or } N$$

where the newton (N) is a derived unit.

3.1 Physical constants

Some physical quantities in nature are "constant" and their values can be found at the NIST website. For example, the speed of light, the mass of the proton, the charge of the electron, and so on.

3.2 Unit Prefixes

Once we've defined a unit, it is easy to introduce larger and smaller units. For example, $1/1000^{\text{th}}$ of a meter can be abbreviated as 1 mm. A thousand meters can be abbreviated as 1 km. A complete set of prefixes can be found at the NIST web site

4 Unit Consistency and Conversions

An equation must always be dimensionally consistent.

$$\text{distance [meters]} = \text{velocity [meters/second]} \times \text{time [seconds]}$$

Exercise 1.5 The most powerful engine available for the classic 1963 Chevrolet Corvette Sting Ray developed 360 horsepower and had a displacement of 327 cubic inches. Express this displacement in liters (L) by using only the conversions 1 L = 1,000 cm^3 and 1 in. = 2.54 cm.

5 Uncertainty and Significant Figures

5.1 Uncertainties

Whenever a physical quantity is measured, it should be quoted in the following manner:

$$\ell \;=\; (4.56 \pm 0.04)\,\text{cm} \qquad = \qquad x \pm \delta x$$

where x is the *mean* or "best" value that can be determined, and δx is the *uncertainty* associated with the measurement.

The **accuracy** of a measured quantity is how close it is to the true value.

The **precision** of a measurement refers to the quality of the measurement. This is usually quoted by calculating the relative uncertainty:

$$\%\text{ relative uncertainty} \qquad \frac{\delta x}{x} \times 100\% \;=\; \frac{0.04}{4.56} \times 100\% \;=\; 0.88\%$$

5.2 Significant Digits

Upon making a measurement, it is important to record the correct number of significant digits. Furthermore, it is important to maintain the correct number of significant digits in subsequent calculations. The number of significant digits quoted in the final answer is one of the motivating reasons for developing *scientific notation*. Here is a website describing some of the general rules for maintaining the correct number of significant digits.

6 Estimates and Orders of Magnitude

Once we know the precision of the numbers we are manipulating, we can make educated guesses (or estimates) of our results. Whenever we perform multiple calculations on a calculator, we hope that our final result is *close* the our "best guess." If not, we should re-do our calculation or rethink how we came up with **order-of-magnitude estimate**.

Example: How thick are the pages in your textbook?

7 Vectors and Vector Addition

A **scalar** has magnitude only.

A **vector** has both magnitude and direction.

7.1 Definition of a Vector

The simplest vector $\rightarrow$ the **displacment vector**

Vector notation: $\vec{A}$ (with an arrow over the top) or $\mathbf{A}$ (boldface)

The **negative** of a vector

Vectors that are **parallel** or **antiparallel**

The magnitude of a vector: (Magnitude of $\vec{A}$) $=$ A or $= |\vec{A}\,|$

7.2 Vector Addition (graphically)

Suppose a *displacement* described by $\vec{C}$ is the result of two displacement vectors, vector $\vec{A}$ followed by vector $\vec{B}$. How can we graphically represent the sum of these two vectors?

$$\vec{C} = \vec{A} + \vec{B}$$

How can we graphically represent the difference between two vectors?

$$\vec{D} = \vec{A} - \vec{B}$$

8 Components of Vectors–numerical addition of vectors

Any vector on the x-y plane can be reduced to the sum of two vectors, one along the x axis, and the other along the y axis.

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where the magnitude of $\vec{A}_x$ is $A\cos\theta$, and the magnitude of $\vec{A}_y$ is $A\sin\theta$. This is sometimes written as

$$\begin{aligned} \text{magnitude of } \vec{A}_x &= A_x = A\cos\theta \\ \text{magnitude of } \vec{A}_y &= A_y = A\sin\theta \end{aligned}$$

Likewise, if we know A_x and A_y, then we can calculate the magnitude and direction of the vector from the following equations:

$$\begin{aligned} A &= \sqrt{A_x^2 + A_y^2} \qquad \text{and} \\ \theta &= \arctan\left(\frac{A_y}{A_x}\right) \end{aligned}$$

8.1 Questions and Answers regarding vectors

9 Unit Vectors

9.1 Unit vectors in the Cartesian coordinate system

Define the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Now, we can write a vector $\vec{A}$ in terms of the *unit vectors.*

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

Exercise 1.33 A disoriented physics professor drives 3.25 km north, then 2.20 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.

10 Product of Vectors

When considering the product of two vectors, there are two kinds of results one can obtain, either a *scalar* or a *vector.*

10.1 Scalar Product

$$\vec{A} \cdot \vec{B} = A\,B\,\cos\phi \qquad \text{(scalar dot product)}$$

Do some examples.

Exercise 1.43 For the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. E1.24, find the scalar products a) $\vec{A} \cdot \vec{B}$; b) $\vec{B} \cdot \vec{C}$; c) $\vec{A} \cdot \vec{C}$.

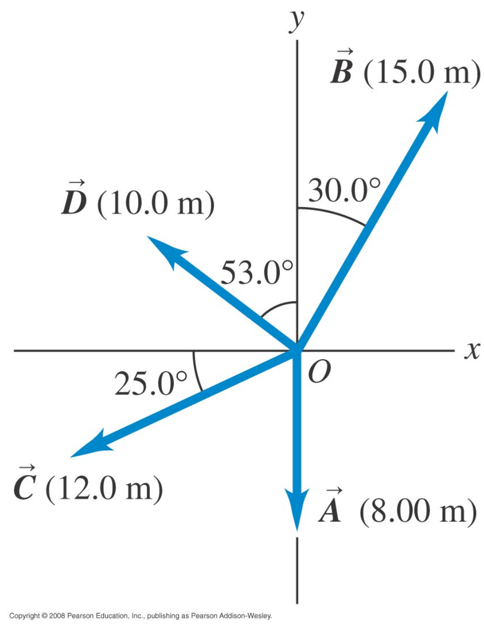

Figure 2: Figure E1.24 from University Physics 14th edition

10.2 Vector Product

$$\vec{C} = \vec{A} \times \vec{B} \qquad \text{(vector cross product)}$$

$$\text{The magnitude of } \vec{C} = A\,B\,\sin\phi$$

Discuss the **right-hand rule** and **right-handed** coordinate systems.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$$

Do some examples.

Exercise 1.47 For the two vectors $\vec{A}$ and $\vec{D}$ in Fig. E1.24 (see the figure above),
a) find the magnitude and direction of the vector product $\vec{A} \times \vec{D}$;
b) find the magnitude and direction of $\vec{D} \times \vec{A}$.

Exercise 1.80 A cube is placed so that one corner is at the origin and three edges are along the x-, y-, and z-axes of a coordinate system (**Fig. P1.80)**. Use vectors to compute (a) the angle between the edge along the z-axis (line ab) and the diagonal from the origin to the opposite corner (line ad), and (b) the angle between the line ac (the diagonal of a face) and line ad.

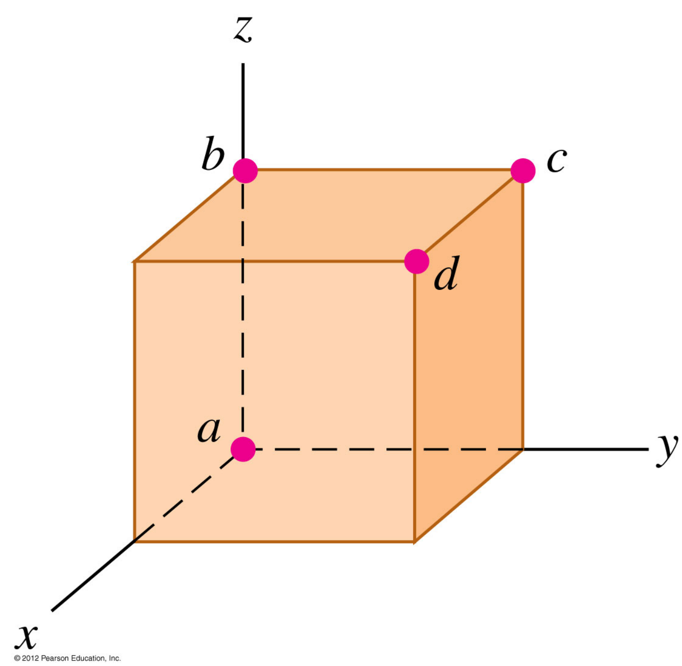

Figure 3: Figure P1.80 from University Physics 14th edition

10.3 Unit Vectors

Let's do an example of a vector cross-product using two 3-dimensional vectors:

$$\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k} \qquad \text{and} \qquad \vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{\imath}(-3-2) + \hat{\jmath}(1-6) + \hat{k}(4+1) \tag{1}$$

$$\vec{C} = \vec{A} \times \vec{B} = \left(-5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}\right) \tag{2}$$

We can also write this vector in a "graphical" representation $\vec{C} = |\vec{C}|\,\hat{C}$ where $\hat{C}$ is a unit vector pointing in the direction of $\vec{C}$. Converting $\vec{C}$ into a *unit vector* is a straight-forward process. Multiply $\vec{C}$ by a constant a such that a **expands** or **shrinks** $\vec{C}$ into a vector of unit length.

$$\hat{C} \;=\; a\,\vec{C} \;=\; a(-5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}) \tag{3}$$

From the definition of a unit vector, we know that $\hat{C} \cdot \hat{C} = 1$. So,

$$\hat{C} \cdot \hat{C} \;=\; a^2(25 + 25 + 25) \;=\; 1 \tag{4}$$

Solving for a, and taking the *positive* square-root because we don't want to *flip* the direction of $\hat{C}$, we find that $a = 1/\sqrt{75} = 1/(5\sqrt{3})$. Now we can write $\vec{C}$ in its "graphical" representation as:

$$\vec{C} \;=\; |\vec{C}|\,\hat{C} \tag{5}$$

where $|\vec{C}| = \sqrt{25 + 25 + 25} = 5\sqrt{3}$, and $\hat{C} = \frac{1}{\sqrt{3}}(-\hat{\imath} - \hat{\jmath} + \hat{k})$.

You can check that $\hat{C}$ is a *unit vector* by confirming that $\hat{C} \cdot \hat{C} = 1$.

10.4 Questions and Answers on Vector Products

11 Challenge Problem

1.101 **Navigating the Big Dipper.** All the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. One light year (ly) is 9.461×10^{15} m.

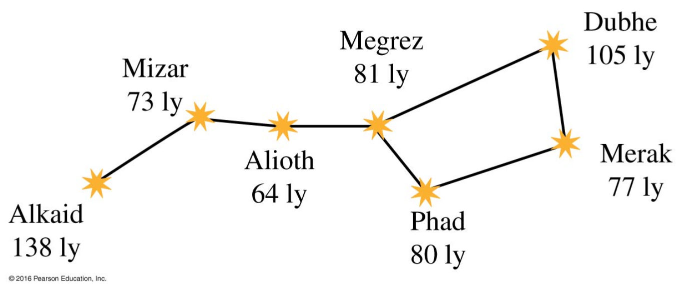

Figure 4: University Physics 14^{th} edition figure P1.91 for problem 1.91

a.) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light years from Alkaid to Merak.

b.) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Extra Problems

Exercise A: A *light-year* is the distance light travels in one year (at a speed = 2.998×10^8 m/s). *(a)* How many meters are the in 1.00 light-year? *(b)* An astronomical unit (AU) is the average distance from the Sun to the Earth, 1.50×10^8 km. How many AUs are there in 1.00 light-year? *(c)* What is the speed of light in AU/hr?

Exercise B: The diameter of the Moon is 3480 km. *(a)* What is the surface area of the Moon? *(b)* How many times larger is the surface area of the Earth?

Exercise C: Show that the following combination of the three fundamental constants of nature (h, G, and c) form a quantity with the dimensions of time:

$$t_p = \sqrt{\frac{Gh}{c^5}}\,.$$

This quantity, t_p, is called the *Planck time* and is thought to be the earliest time, after the creation of the Universe, at which the currently known laws of physics can be applied.

Exercise D: Given the following three vectors:
$\vec{A} = 4.0\hat{\imath} - 8.0\hat{\jmath}$,
$\vec{B} = \hat{\imath} + \hat{\jmath}$, and
$\vec{C} = -2.0\hat{\imath} + 4.0\hat{\jmath}$,

(a) Calculate $\vec{A} + \vec{B} + \vec{C} = ?$, and

(b) $\vec{A} - \vec{B} + \vec{C}$=?