e/m Charge-to-Mass Ratio Lab

Dr. Darrel Smith[1]
Physics Department
Embry-Riddle Aeronautical University

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The purpose of this experiment is to measure the charge-to-mass ratio of the electron, e/m. In this experiment, electrons are accelerated and execute circular motion perpendicular to a homogeneous magnetic field produced by a pair of Helmholtz coils. The accelerating voltage, and strength of the magnetic field can be used to determine the e/m ratio.

I. BACKGROUND

Because of the very small mass of the electron (9.11×10^{-31} kg), the electron has the largest charge-to-mass ratio of any system or elementary particle ($e/m = 1.76 \times 10^{11}$C/kg. As a result, large accelerations can be produced with only a modest electric field ($\vec{E}$). By applying **Newton's 2$^{\text{nd}}$ law**, the relationship between the acceleration and the charge-to-mass ratio can be shown.

$$\sum F_{\text{ext}} = ma \tag{1a}$$

$$qE = ma \tag{1b}$$

$$a = \frac{e}{m} E \tag{1c}$$

In the early 1900's, electrons were known as *cathode rays* and the only known property for these particles was the e/m ratio. The ability to steer electrons required knowledge of only two parameters, the electric field E, and the charge-to-mass ratio e/m.

II. THE EXPERIMENT

In this experiment, the electrons are momentarily accelerated from rest (using an electric field) perpendicular to a magnetic field produced by a pair of Helmholtz coils. As a result the electrons move in a circular path described by **Newton's 2$^{\text{nd}}$ law**.

$$F = qvB = \frac{mv^2}{r} \tag{2a}$$

$$\frac{e}{m} = \frac{v}{rB} \tag{2b}$$

As the electrons are accelerated through the electric field, **conservation of energy** can be applied to determine the velocity:

$$K = U \qquad \frac{1}{2} mv^2 = eV \tag{3a}$$

$$v = \sqrt{\frac{2eV}{m}} \tag{3b}$$

where v is the velocity, V is the accelerating potential (in volts), and e is the charge of the electron. Combining Eqs. 2b and 3b, one can calculate the equation for e/m.

$$\frac{e}{m} = \sqrt{\frac{2V}{r^2 B^2}} \tag{4}$$

However, we can use the following linear relationships, $B = kI$ and $V = c\,I^2$, to obtain the final equations for e/m.

$$\frac{e}{m} = \frac{2\,c}{r^2\,k^2} \tag{5}$$

where

$k = \mu_0 \left(\frac{4}{5}\right)^{3/2} \frac{n}{R}$	from the Biot-Savart law,
$R = 15.0$ cm	Helmholtz coil radius, and
$n = 130$ turns	for a single coil.

III. THE EQUIPMENT

A vacuum tube with rarified hydrogen gas is positioned between the two Helmholtz coils. A power supply box containing three power supplies (5A, 10mA, and 50mA) is connected to the experiment. The 5A supply is for the Helmholtz coils that produce the magnetic field for this experiment while the third supply (50 mA) is the high voltage supply for accelerating the electrons from rest to their final velocity. More details about the magnetic field produced by Helmholtz coils can be found at the following website: `http://physicsx.pr.erau.edu/helmholtzcoils/`.

IV. PROCEDURE

You will find more information regarding the procedures on my PS315 webpage. It contains an instruction sheet, a data analysis sheet, and a Classical e/m instruction sheet (more for historical interest). The first instruction sheet describes the K.T. Bainbridge e/m experiment, the one we will be using.

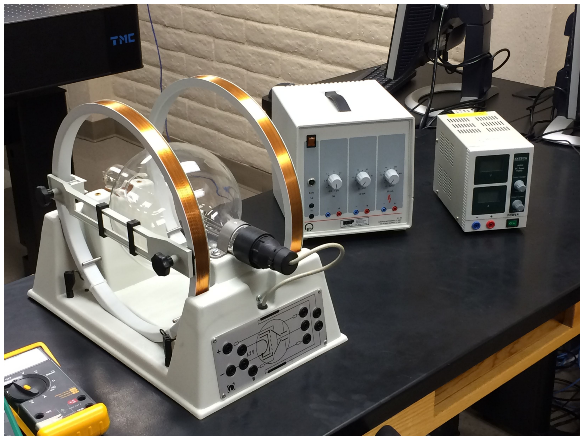

FIG. 1. The charge-to-mass experiment is shown in the figure above along with the triple power supply. A spare high current (low voltage) supply is shown to the right. The triple power supply should be sufficient.

A. Instruction Sheet

As you read through the instruction sheet, become familiar with the names attached to the critical parts of the experiment. Also read the highlighted *Safety Notes* on the first page regarding the use of high voltage in this experiment.

B. Data Analysis

This leaflet should be read with great interest and understood. It describes the theory and guiding principles involved in this experiment. **It also describes how one should go about collecting the data for this experiment.** The experimental apparatus you are using was built in Germany and you will not see the words *charge-to-mass* used in this document. Instead, they refer to e/m as the "specific charge." Also, they use the symbol U to refer to the electrostatic potential (volts). In the United States, we tend to use V as the electrostatic potential. So, please make those two connections.

Again, please read the "Safety Notes" found on page 2 of this leaflet. In particular, do not touch the Helmholtz coils while this experiment is in operation.

On page 3 is described a procedure for focussing the electron beam by varying the voltage on the Wehnelt-cylinder from 0 ... 10 V until it leads to a narrow, well defined beam. In previous years, the students have commented that this procedure produces little, if any, difference in beam definition. However, give it a try.

As you can see from Fig. 6 on page 4, you will want to collect data with the circular orbit at a constant diameter (8 cm) for a range of voltages U (300 V $\rightarrow$ 200 V) while recording the magnetic current I (in amperes).

Please take note of the comments made on page 2 of this leaflet. There are two ways to calculate the magnetic field. Go ahead and use Eqs. 7, 8, and 9 along with the other physical parameters recorded on this page in order to calculate the magnetic field as a function of the current (i.e., $B = k\,I$). For your information, we do have a magnetometer to measure the actual magnetic fields if you are interested.

C. Classic Experiment

This instruction sheet did not come with this apparatus. However, it describe our e/m experiment (what they call Experiment 2), and another experiment (Experiment 1) that uses crossed electric and magnetic fields to measure the e/m ratio. I found this leaflet to be a useful resource as you search for other reading material regarding this lab. It is always insightful to find other explanations of the same experiment to help build your understanding of what you are doing. I found the sections on "Historical Background" and "Experiment 2" to be informative.

V. IMPORTANT CONSIDERATIONS

- **Be careful.** You will be using high voltage in this experiment.
- Have your lab assistant or professor "check" your wiring before turning on the voltage supplies.
- Low voltages can also present a health hazard. While the voltage supply to the magnet does not exceed 7.5 volts, it produces a curent up to 5A.
- The copper wires in the Helmholtz coils appear to be bare, but they are covered with a thin, clear polyurethane coating. This insulation should provide adequate protection; in any case you should still avoid touching the wires in the Helmholtz coils.
- Again, if you are unsure about operating the apparatus, please ask for assistance. The equipment is moderately expensive; however, your health and well-being are more important.
- **Don't forget** to do your error analysis. You will need to do the error analysis of a straight line fit, and afterwards propagate the uncertainties correctly to quote your final answer in the following format:

$$e/m = \left(\frac{e}{m}\right)_{\text{fitted}} \pm \delta\left(\frac{e}{m}\right)$$

VI. HOW TO TREAT ERROR BARS IN BOTH THE X AND Y DIRECTIONS

In this lab you are asked to plot your data as shown in Fig. 6 in the Data Analysis leaflet. In this figure the potential U is plotted as a function of I^2 resulting in the linear trend shown in Fig. 2. While most fitting programs fit data having error bars in the *vertical* direction, they don't include the *horizontal* error bars when determining the errors in the fitted parameters. So, the question naturally arises, "How can one include the *horizontal* error bars as part of the fit?"

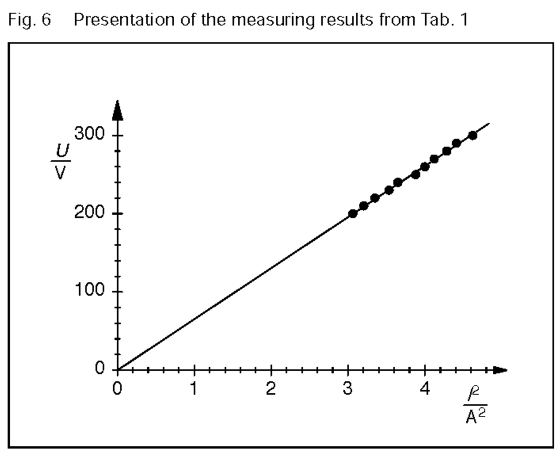

FIG. 2. A plot of the voltage versus the current squared. This is from Fig. 6 found in the Data Analysis leaflet.

Let's imagine that we have error bars as shown in Fig. 3. Let's define a procedure where the error bar in the x direction (σ_x) is reduced to zero, thus expanding the error bar in the y direction to a new value (σ'_y). In order

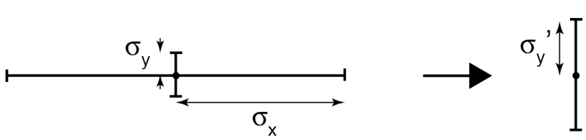

FIG. 3. Combine both error bars in the x and y direction (σ_x and σ_y) into a single error bar in the y direction (σ'_y).

to convert the *horizontal* and the *vertical* error bars into a single error bar (σ'_y), we need to know the relationship between U and I^2. In this case, we will use the obvious relationship $U = c\,I^2 + \text{const}$ where c is the slope of the straight-line fit. When determining the parameters of the straight-line fit, the results should report the fitted value of the slope (c) as well as the uncertainty of the slope (δc).

With this information in hand, we can now determine the relationship between the two error bars (σ_x, σ_y). Using the fitting function $U = c\,I^2 + \text{const}$, we see that:

$$\delta U \;=\; c\; 2I\,\delta I$$

or

$$\delta U \;=\; U\,\frac{2\,\delta I}{I}$$

where δI is the uncertainty in the current, and U is the potential at that particular current. Adding this correction to the already existing error bar in the *vertical* direction we find the new error bar in the y direction:

$$\sigma'_y \;=\; \sqrt{(\sigma_y)^2 + (\delta U)^2} \tag{6}$$

The new error bar (σ'_y) is larger than the original error bar in the y direction (σ_y), and combines the original error bars (σ_x, σ_y) into a single error bar. The size of the new error bar (σ'_y) is not the same for every point. Make sure the new error bars (σ'_y) are calculated for each data point. By doing this, we have replaced the old fitting weights $w_i = 1/\sigma_i^2$ with new weights $w'_i = 1/\sigma_i'^2$. In this experiment, the error bars (σ'_i) will all be different, thus the weights for each of the data points are different.

At this point, the fit to $U = c\,I^2 + \text{const}$ should be redone with the new error bars included in the fit. While the slope c may not change by much, there should be a definite increase in the uncertainty in the slope (δc).

In this lab, the slope c is related to the following quantities:

$$c \;=\; \frac{e}{m_e}\,\frac{1}{2}\,r^2\,k^2$$

where k can be found in the *Data Analysis* sheet on my website, $k = \mu_o(4/5)^{3/2}n/R$ determined from the Biot-Savart law. See my website regarding Helmholtz coils. Solving this equation for the *charge-to-mass* ratio, we find:

$$\frac{e}{m_e} \;=\; \frac{2\,c}{r^2\,k^2}$$

Using this equation, one can complete the error analysis using the familiar *propagation of uncertainties* technique:

$$\delta\left(\frac{e}{m_e}\right) \;=\; \left(\frac{e}{m_e}\right)\sqrt{\left(\frac{\delta c}{c}\right)^2 + \left(\frac{2\,\delta r}{r}\right)^2}$$

where we have assumed that the uncertainty in k (δk) is very small and close to *zero*.