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Chapter 15 In-Class Problems

Ex. 1 The speed of sound in air at 20°C is 344 m/s.

a.) $\lambda = ?$ if $f = 784$ Hz

$$\lambda = \frac{v}{f} = \frac{344\ m/s}{784\ /s} = \underline{0.439\ m}$$

b.) $f \rightarrow 2f$ (one octave)

$$\lambda = \frac{v}{2f} = \frac{344\ m/s}{2(784\ /s)} = \underline{0.219\ m}$$

Ex. 8 A certain transverse wave is described by

$$y(x,t) = (6.50\,mm)\cos\left[2\pi\left(\frac{x}{\underbrace{28.0cm}_{\lambda}} - \frac{t}{\underbrace{0.0360s}_{T}}\right)\right]$$

a.) Amplitude $= \underline{6.50cm}$

b.) Wavelength $\rightarrow \lambda = \underline{28.0\ cm}$

c.) Frequency $\rightarrow f = \frac{1}{T} = \frac{1}{0.0360s} = \underline{27.8\ Hz}$

d.) Speed of Propagation $\rightarrow v = f\lambda = (27.8\ s^{-1})(28.0cm) = \underline{778\ cm/s}$

e.) Direction of Propagation - To the Right $\boxed{L \rightarrow R}$ + x-direction

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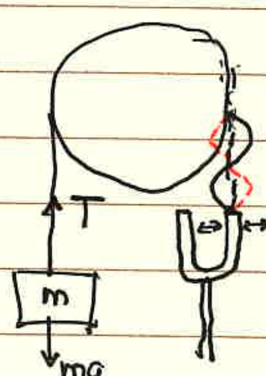
Ex. 15 One end of a horizontal rope is attached to a prong of an electronically driven tuning fork $\rightarrow f = 120$ Hz

F = force = mg μ = linear mass density = 0.0480 kg/m

a.) $v = f\lambda$ or $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(1.50\ kg)(9.80\ m/s^2)}{0.0480\ kg/m}} = \underline{17.5\ \frac{m}{s}}$

b.) $\lambda = ?$ $\lambda = \frac{v}{f} = \frac{17.5\ m/s}{120\ /s} = \underline{0.146\ m}$



c.) How would your answers change in parts (a) & (b), if the mass were increased $\rightarrow$ 3.00 kg?

$v' \rightarrow v\sqrt{2} = \underline{24.7\ m/s}$ faster

$\lambda' \rightarrow \frac{v'}{f} = \frac{24.7\ m/s}{120\ /s} = \underline{0.206\ meters}$ longer

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Ex. 22 A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. $f = 120$ Hz $A = 1.6$ mm

a.) Average Power carried by the wave → $P_{Av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$$P_{Av} = \frac{1}{2}\sqrt{\underbrace{\left(\frac{3\times10^{-3}\,\text{kg}}{0.80\,\text{m}}\right)}_{\mu}\underbrace{25.0\,\text{N}}_{F}}\;\left(2\pi\,(120\,\text{s}^{-1})\right)^2\left(1.6\times10^{-3}\,\text{m}\right)^2$$

$$\boxed{P_{Av} = 0.223\ \text{Watts}}$$

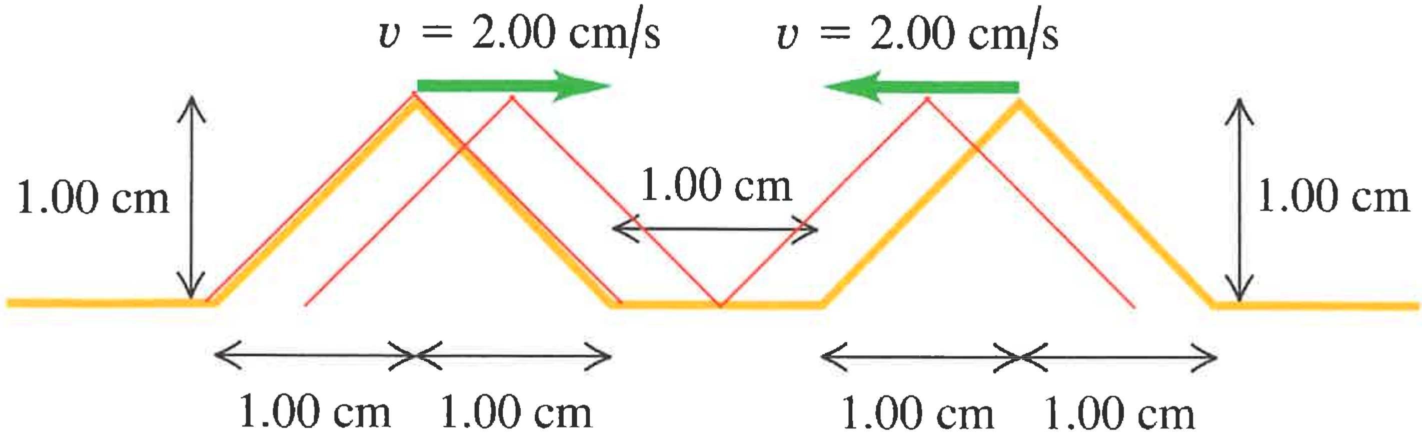
b.) What happens to P_{Av} if the $A' = \frac{1}{2}A$

$$P'_{Av} = \frac{1}{4}P_{Av} = 0.056\ \text{W}$$

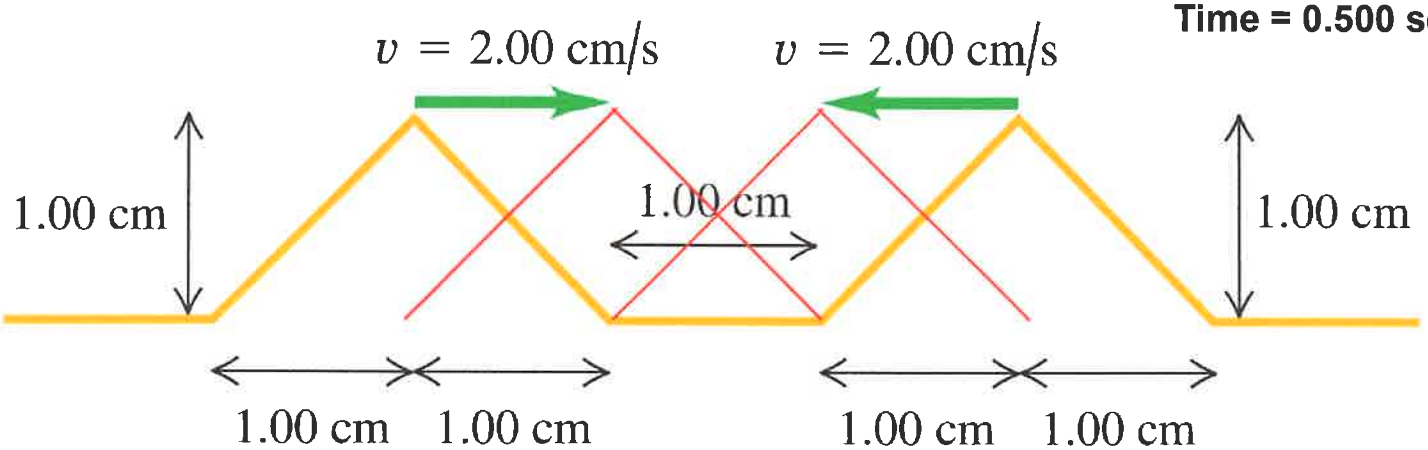
Ex. 30 Interference of Triangular Pulses

Sketch the shapes at $t = 0.250$ sec, 0.500 sec, 0.750 sec, 1.00 sec, 1.250 sec

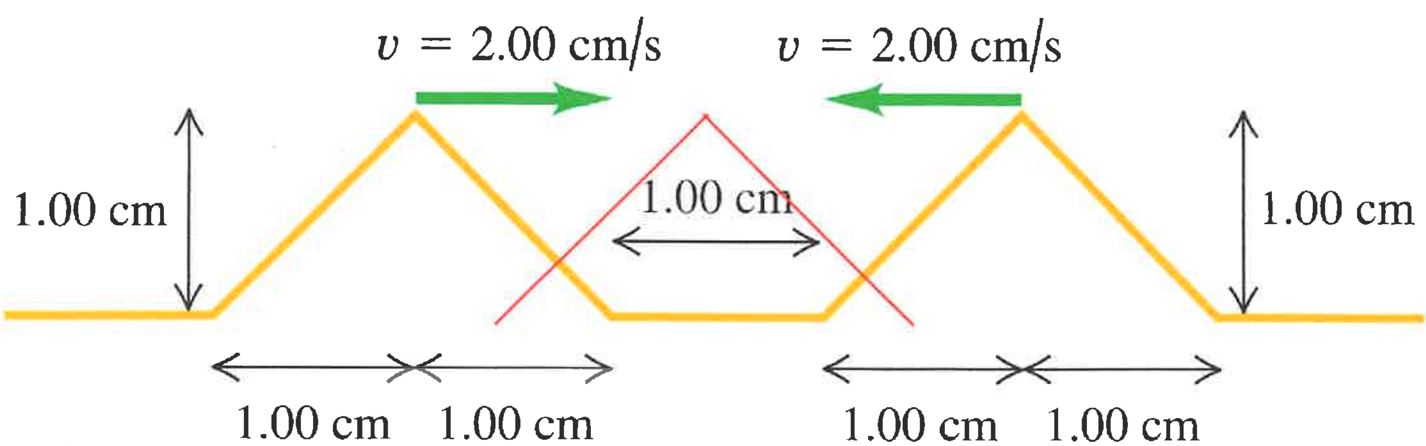
Time = 0.250 sec.



Time = 0.500 sec.



Time = 0.750 sec.



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Ex. 35 Standing waves on a wire are described by Eq. 15.28

$A_{sw} = 2.50$ mm $\qquad y(x,t) = A_{sw}\sin(kx)\sin(\omega t)$

$\omega = 942$ rad/s

$k = 0.750\pi$ rad/m

The left end of the wire is at $x = 0.00$ m. Where are the (a) nodes and (b) antinodes located?

0.750 rad/m

(a.) nodes occur at $kx = 0, \pi, 2\pi, \ldots$

$$x_1 = \frac{0 \text{ rad.}}{0.750\pi \text{ rad/m}} = 0 \text{ meters}$$

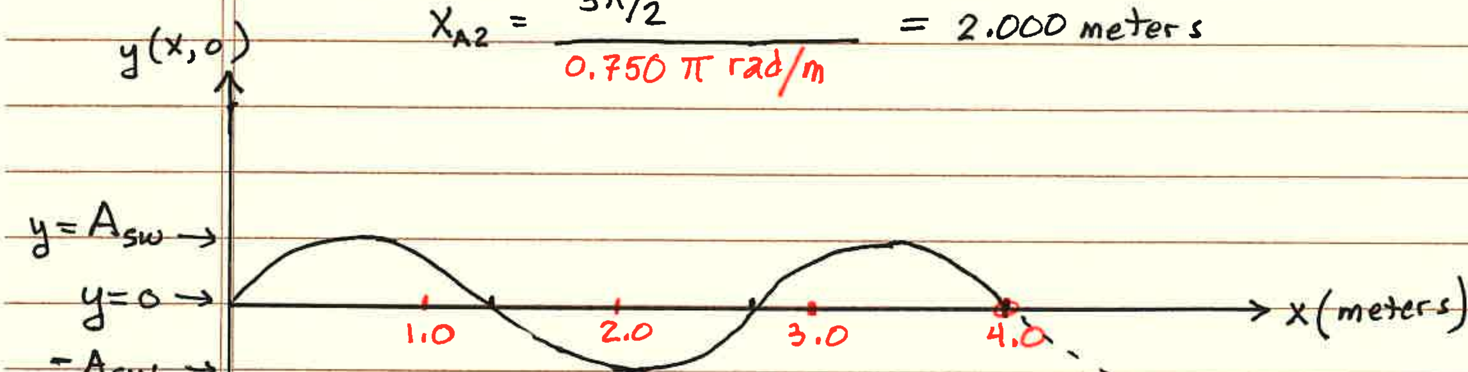
$$x_2 = \frac{\pi \text{ rad.}}{0.750\pi \text{ rad/m}} = 1.\overline{333} \text{ meters}$$

$$x_3 = \frac{2\pi \text{ rad}}{0.750\pi \text{ rad/m}} = 2.\overline{666} \text{ meters}$$

(b.) antinodes occur at $kx = \pi/2, \; 3\pi/2, \; 5\pi/2, \ldots$

$$x_{A1} = \frac{\pi/2}{0.750\pi \text{ rad/m}} = 0.\overline{666} \text{ meters}$$

$$x_{A2} = \frac{3\pi/2}{0.750\pi \text{ rad/m}} = 2.000 \text{ meters}$$



$x_{nodes} = 0, \; 4/3, \; 8/3, \; 4, \; 16/3, \ldots$ etc. meters

$x_{Antinodes} = 2/3, \; 2, \; 10/3, \; 14/3, \; 18/3, \ldots$ etc. meters

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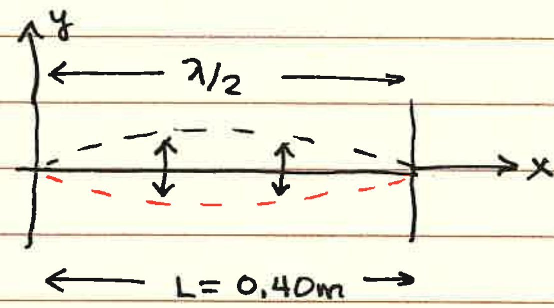
Ex. 38 A piano tuner stretches a steel piano wire with a tension of 800 N.

$L = 0.400\ m \qquad m = 3.00\ g$

a.) fundamental frequency



$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{800\ N}{3.00\times10^{-3}/0.400}}$$

$v = 326.6\ m/s$

$$\boxed{v = f\lambda} \qquad f = \frac{v}{\lambda} = \frac{326.6\ m/s}{0.800\ m} = \underline{\underline{408.2\ Hz}} = f_1$$

$$\underline{\text{or}} \text{ use } f_n = \frac{nv}{2L} \Rightarrow f_1 = \frac{v}{2L} = 408.2\ Hz$$

b.) What is the highest harmonic a person could hear who is capable of hearing up to 10,000 Hz?

$$\Rightarrow \quad nf_1 = 10{,}000\ Hz \qquad n = \frac{10{,}000\ Hz}{408.2\ Hz} = 24.495$$

So, the highest harmonic heard would be the $\underline{\underline{24^{th}\ \text{harmonic}}}$.

Ex. 47 Guitar String: One of the 63.5 cm-long strings of an ordinary guitar $\rightarrow B_3\ (f_1 = 245\ Hz)$

a.) find the transverse speed of the waves on this string

$$V = f\lambda = f_1(2L) = (240\ s^{-1})(2 * 0.635\ m) = \underline{\underline{304.8\ m/s}}$$

b.) If the tension in the string is increased by 1%, $f_1' = ??$

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Ex. 47 Cont'd

$f' = v'\lambda$ $\lambda \rightarrow$ remains the same. We assume that the guitar string does not get stretched.

$$f' = \sqrt{\frac{F'}{\mu}}\,\lambda \iff f = \sqrt{\frac{F}{\mu}}\,\lambda$$

① $\boxed{f' = f + df}$

Using Calculus $\Rightarrow$ $df = \frac{\lambda}{\sqrt{\mu}} \frac{1}{2} F^{-1/2}\, dF$ $\quad \frac{df}{f} = \frac{\frac{\lambda}{\sqrt{\mu}} \frac{1}{2} F^{-1/2}\, dF}{\frac{\lambda}{\sqrt{\mu}} F^{1/2}} = \frac{1}{2}\frac{dF}{F}$

However $\rightarrow$ $\frac{dF}{F} = 0.01$ (1% percent)

So, $\frac{df}{f} = \frac{1}{2}\frac{dF}{F}$ $\quad df = f\,\frac{1}{2}(0.01) = 1.225$ Hz

245 Hz

From Eq. ① $f' = 245\text{ Hz} + 1.2\text{ Hz}$ $\boxed{f' = 246\text{ Hz}}$

c.) If the speed of sound in the surrounding air is 344 m/s, $f = ??$ and $\lambda = ??$

in air $\rightarrow$ $f = 245$ Hz, the same as the frequency of the B_3 string

in air $\rightarrow$ $\lambda = \frac{v_{air}}{f} = \frac{344\text{ m/s}}{245\text{ /s}} = 1.404$ meters, which is different from the wavelength of the string $\rightarrow$ $\lambda_{string} = 2L = 1.270$ meters

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Ex. 46 A transverse wave on a rope is given by:

$$y(x,t) = (0.75\text{ cm}) \cos\left[\pi\left((0.400\text{ cm}^{-1})x + (250\text{ s}^{-1})t\right)\right]$$

a.) Find A, T, f, λ, v (the speed of propagation)

Compare the above equation to $y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$

$y(x,t) = A\cos(kx - \omega t)$

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Ex. 46 cont'd $\boxed{A = 0.75\ cm}$ $2\pi f = (250\ s^{-1})\pi \rightarrow \boxed{f = 125\ Hz}$

$T = \frac{1}{f} = 8.00 \times 10^{-3}\ sec$ $\boxed{T = 8.00\ ms}$

$\lambda \rightarrow \frac{2\pi}{\lambda} = \pi(0.400\ cm^{-1})$ $\lambda = \frac{2}{(0.400\ cm^{-1})}$ $\boxed{\lambda = 5.00\ cm}$

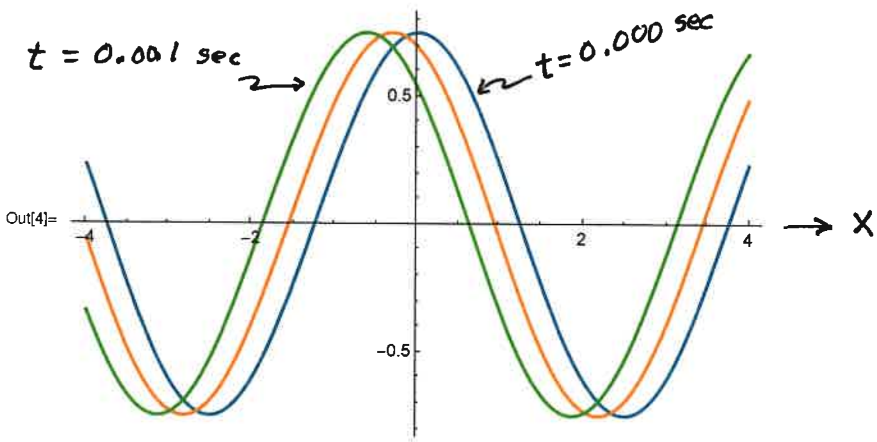
$v = f\lambda \rightarrow V = (125\ Hz)(5\ cm) = 6.25\ m/s$ $\boxed{V = 6.25\ m/s}$

b.)

```
In[1]:= y[x_, t_] = 0.75 Cos[π (0.400 x + 250. t)]
Out[1]= 0.75 Cos[π (250. t + 0.4 x)]

In[4]:= Plot[{y[x, 0], y[x, 0.0005], y[x, 0.001]}, {x, -4, 4}]
```

Out[4]=



c.) The wave is traveling in the <u>−x direction</u>

d.) $\mu = 0.0500\ kg/m$ $T = ?$ $v = \sqrt{\frac{T}{\mu}}$ $T = \mu v^2 = (0.0500)(6.25)^2$

$\boxed{T = 1.95\ N}$

e.) $P_{AV} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\sqrt{(0.0500)(1.95)}\,(2\pi(125\ Hz))^2(0.0075\ m)^2$

$\boxed{P_{AV} = 5.42\ W}$

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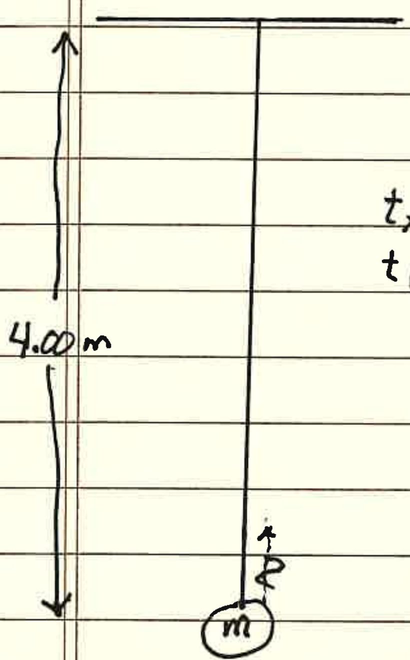
Ex. 54

You are exploring a newly discovered planet. The radius of the planet is 7.10×10^{7} m.



$t_x = 0.0655\ s$ for tension $= mg_x$

$t_E = 0.0360\ s$ for tension $= mg_E$ $\quad g_E = 9.8 \frac{m}{s^2}$

$$V = \sqrt{T/\mu} \qquad V_x = \sqrt{\frac{mg_x}{\mu}} \qquad V_E = \sqrt{\frac{mg_E}{\mu}}$$

$$t_x = \frac{4m}{V_x} = 4m\sqrt{\frac{\mu}{mg_x}}$$

$$t_E = \frac{4m}{V_E} = 4m\sqrt{\frac{\mu}{mg_E}}$$

$$\frac{t_E}{t_x} = \frac{4m\sqrt{\frac{\mu}{mg_E}}}{4m\sqrt{\frac{\mu}{mg_x}}} = \sqrt{\frac{g_x}{g_E}} \qquad g_x = g_E\left(\frac{t_E}{t_x}\right)^2$$

$$g_x = 9.8\ m/s^2\left(\frac{0.0360\ s}{0.0655\ s}\right)^2 = \underline{2.960\ m/s^2}$$

$$g_x = \frac{GM}{R^2} \qquad M = \frac{g_x R^2}{G} = \frac{2.960\ m/s^2\ (7.10 \times 10^{7}\ m)^2}{6.67 \times 10^{-11}\ N \cdot m^2/kg^2}$$

$$\boxed{M = 2.24 \times 10^{26}\ kg}$$

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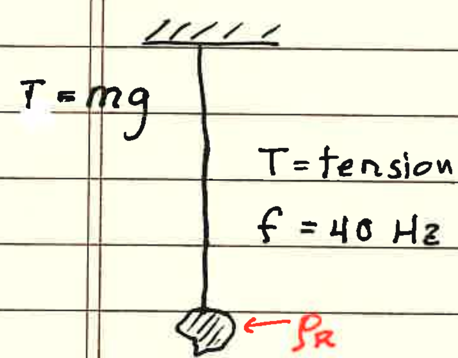
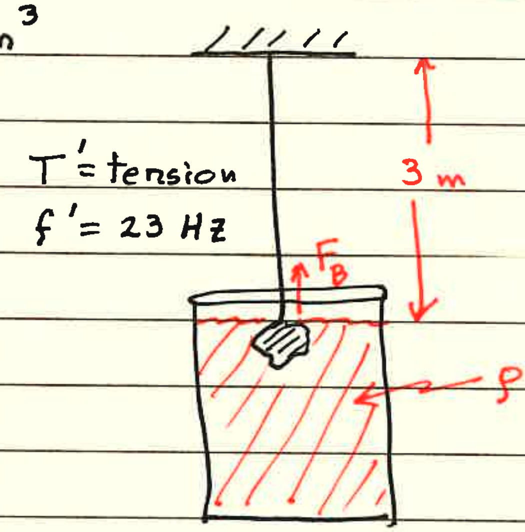
Ex. 69 A large rock that weighs 164.0 N is suspended from the lower end of a thin wire that is 3.00 m long.

$\rho_R = 3200\ \mathrm{kg/m^3}$





$$T' = mg - F_B = \rho_R V g - \underbrace{\rho V g}_{F_B}$$

$$T = mg = \rho_R V g$$

$$\frac{T'}{T} = \frac{\rho_R V g - \rho V g}{\rho_R V g} = 1 - \frac{\rho}{\rho_R} \qquad \boxed{\frac{T'}{T} = 1 - \frac{\rho}{\rho_R}}$$

$$v = f\lambda = \sqrt{T/\mu} \qquad f\lambda = \sqrt{T/\mu} \qquad f'\lambda = \sqrt{T'/\mu} \qquad \frac{f'}{f} = \sqrt{\frac{T'}{T}}$$

$$\left(\frac{f'}{f}\right)^2 = \frac{T'}{T} = 1 - \frac{\rho}{\rho_R} \qquad \frac{\rho}{\rho_R} = 1 - \left(\frac{f'}{f}\right)^2$$

$$\rho = 3200\ \mathrm{kg/m^3}\left(1 - \left(\frac{23\ \mathrm{Hz}}{40\ \mathrm{Hz}}\right)^2\right) = \boxed{2142\ \mathrm{kg/m^3}}$$