

In-Class Homework Problems Ch. 14 Periodic Motion.

DATE	
TOPIC	①

Ex. 3

The tip of a tuning fork goes through 440 oscillations in 0.500 sec.

Find ω, T . $\omega = 2\pi f = 2\pi \left(\frac{440 \text{ vibrations}}{0.500 \text{ sec}} \right)$
 $f = 880 \text{ Hz}$

$\omega = 5.53 \times 10^3 \text{ rad/s}$

$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{1}{f} = \frac{1}{880 \text{ Hz}} = 1.14 \times 10^{-3} \text{ s}$
 $T = 1.14 \times 10^{-3} \text{ s}$ or 1.14 ms (milliseconds)

Ex. 9

When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, ... $f = 6.00 \text{ Hz}$

a.) Find the period, $T = \frac{1}{f} = \frac{1}{6.00 \text{ Hz}} \Rightarrow T = 0.1666 \text{ sec}$

b.) $\omega = ?$ $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{0.1666 \text{ sec}}$ $\omega = 37.7 \frac{\text{rad}}{\text{s}}$

c.) $m = \text{mass} = ?$ $\omega^2 = \frac{k}{m}$ $m = \frac{k}{\omega^2} = \frac{120 \text{ N/m}}{(37.7 \text{ rad/s})^2}$

$m = 8.44 \times 10^{-2} \text{ kg}$ or $m = 84.4 \text{ grams}$

Ex. 11... 2 pages later

Ex. 28

A harmonic oscillator has angular frequency ω and amplitude A .

a.) What are "x" and "v" when $KE = PE$ (i.e., $K = U$)
 $K = U = \frac{1}{4}kA^2 = \frac{1}{4}m\omega^2A^2$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

"v" $\rightarrow \frac{1}{2}mv^2 = \frac{1}{4} \frac{m\omega^2A^2}{k}$ $v^2 = \frac{1}{2}\omega^2A^2$ $v = \frac{\omega A}{\sqrt{2}}$

"x" $\rightarrow \frac{1}{2}kx^2 = \frac{1}{4} \frac{m\omega^2A^2}{k}$ $x^2 = \frac{1}{2}A^2$ $x = \frac{A}{\sqrt{2}}$

Magnitude Only

In-Class Homework Problems Ch. 14 Periodic Motion

DATE	
TOPIC	2

EX. 28 cont'd

b.) How often does this occur in each cycle?

Let's assume that the displacement $x(t) = A \cos(\omega t)$

$$t_1 \rightarrow \frac{A}{\sqrt{2}} = A \cos(\omega t) \rightarrow \cos(\omega t) = \frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$t_1 = \frac{\pi}{4\omega}$$

$$t_2 \rightarrow -\frac{A}{\sqrt{2}} = A \cos(\omega t) \rightarrow \cos(\omega t) = -\frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

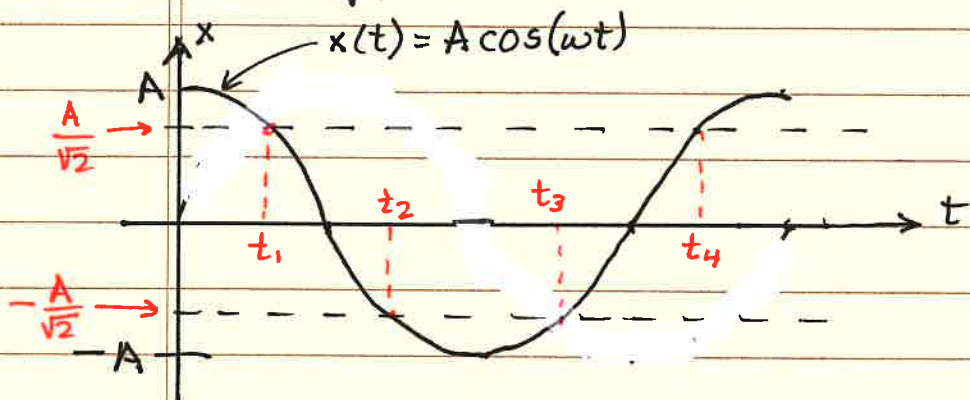
$$t_2 = \frac{3\pi}{4\omega}$$

$$t_3 \rightarrow -\frac{A}{\sqrt{2}} = A \cos(\omega t) \rightarrow \cos(\omega t) = -\frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{5\pi}{4}$$

$$t_3 = \frac{5\pi}{4\omega}$$

$$t_4 \rightarrow \frac{A}{\sqrt{2}} = A \cos(\omega t) \rightarrow \cos(\omega t) = \frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{7\pi}{4}$$

$$t_4 = \frac{7\pi}{4\omega}$$



What is the spacing of these time intervals?

They are all the same $\Delta t = \frac{2\pi}{4\omega}$

$$\Delta t = \frac{\pi}{2\omega} \text{ (seconds)}$$

Note: $\omega = \frac{2\pi}{T}$

So, $\Delta t = \frac{\pi}{2 \times 2\pi} = \frac{T}{4}$

one-quarter of the full period.

c.) At $x = \frac{A}{2}$ $U = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{A}{2}\right)^2 = \frac{1}{8} kA^2 = \frac{1}{4} \left(\frac{1}{2} kA^2\right) = \frac{1}{4} E$

$$K = E - U = \frac{3}{4} E$$

$$U = \frac{1}{4} E \text{ and } K = \frac{3}{4} E$$

DATE	
TOPIC	

Ch. 14 Periodic Motion

Ex. 11

An object is undergoing SHM with a period of 0.900 s and amplitude 0.320 m.

a.) At $t=0$ $x=0.320$ m and $v_x=0$

What is Δt for $x \rightarrow 0.320$ m to $x=0.160$ m.

$$\Delta t = (t - 0) \text{ sec.}$$

In general $x(t) = A \cos(\omega t + \phi)$

In our case, $\phi = 0$ because $x(0) = A$

$$\text{So, } x(t_1) = A \cos(\omega t_1) \rightarrow 0.160 \text{ m} = (0.320 \text{ m}) \cos(\omega t_1)$$

$$\cos(\omega t) = \frac{1}{2} \quad \omega t = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \quad \boxed{t_1 = \frac{\pi}{3\omega}}$$

$$t_1 = \frac{\pi T}{3 \cdot 2\pi} = \frac{T}{6} = \frac{0.900 \text{ s}}{6} = \boxed{0.150 \text{ sec}}$$

b.) What is Δt for $x \rightarrow 0.160$ m to 0.00 m.

$$x(t_2) = A \cos(\omega t_2) \Rightarrow 0.00 \text{ m} = (0.320 \text{ m}) \cos(\omega t_2)$$

$$\Rightarrow \cos(\omega t_2) = 0 \Rightarrow \omega t_2 = \frac{\pi}{2} \text{ radians}$$

$$t_2 = \frac{\pi}{2\omega} = \frac{\pi T}{2 \cdot 2\pi} = \frac{T}{4} = \frac{0.900 \text{ s}}{4} = 0.225 \text{ s}$$

$$\Delta t = t_2 - t_1 = 0.225 \text{ s} - 0.150 \text{ s}$$

$$\boxed{\Delta t = 0.075 \text{ sec}}$$

In-Class Homework Problems Ch. 14 Periodic Motion

Ex. 36

A proud deep-sea fisherman

hangs a 65.0 kg fish from an ideal spring...

$$\Delta x = 0.180 \text{ m}$$

a.) $k = ?$ $F = kx$ "force acting on the spring"
 $mg = kx$ $k = \frac{mg}{x} = \frac{(65.0 \text{ kg})(9.8 \text{ m/s}^2)}{0.180 \text{ m}} = 3.54 \times 10^3 \frac{\text{N}}{\text{m}}$

$$k = 3.54 \times 10^3 \text{ N/m}$$

b.) $T = ?$ Period of oscillation $\omega = \sqrt{k/m}$ $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{65.0 \text{ kg}}{3.54 \times 10^3 \text{ N/m}}} \quad T = 0.851 \text{ sec}$$

c.) $v_{\text{max}} = ?$ $v_{\text{max}} = A\omega = A\left(\frac{2\pi}{T}\right) = 0.05 \text{ m} \left(\frac{2\pi}{0.851 \text{ s}}\right)$

$$v_{\text{max}} = 0.369 \text{ m/s}$$

Ex. 46

A pendulum on mars ... $T_{\text{earth}} = 1.60 \text{ sec.}$

What is its period on the surface of mars? $g_{\text{mars}} = 3.71 \frac{\text{m}}{\text{s}^2}$

$$\omega = \sqrt{\frac{g}{L}} \rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi \sqrt{\frac{L}{g}} \quad T_{\text{mars}} = 2\pi \sqrt{\frac{L}{g_{\text{mars}}}} = \sqrt{\frac{g_{\text{earth}}}{g_{\text{mars}}}}$$

$$T_{\text{earth}} = 2\pi \sqrt{\frac{L}{g_{\text{earth}}}}$$

$$T_{\text{mars}} = T_{\text{earth}} \sqrt{\frac{g_{\text{earth}}}{g_{\text{mars}}}} = 1.60 \text{ s} \sqrt{\frac{9.80}{3.71}} = 2.60 \text{ sec}$$

$$T_{\text{mars}} = 2.60 \text{ sec.}$$

In-Class Homework Problems Ch. 14

DATE

TOPIC

4

Ex. 50

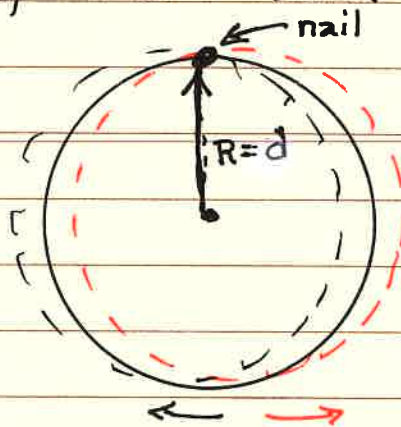
We want to support a thin hoop on a horizontal nail.

$T = 2.0$ seconds $R = ?$

$$\omega = \sqrt{\frac{Mgd}{I}}$$

$$I = I_{cm} + Md^2 = MR^2 + MR^2$$

$$I = 2MR^2$$



$$\omega = \frac{2\pi}{T} = \sqrt{\frac{MgR}{2MR^2}} = \sqrt{\frac{g}{2R}}$$

$$\frac{4\pi^2}{T^2} = \frac{g}{2R} \Rightarrow R = \frac{gT^2}{8\pi^2} = \frac{(9.8 \text{ m/s}^2)(2.0 \text{ s})^2}{8\pi^2}$$

$$R = 0.496 \text{ m}$$

Ex. 57

An unhappy 0.300 kg rodent, moving on the end of a spring... $k = 2.50 \text{ N/m}$ with a damping force $F_x = -bv_x$

a.) $b = 0.900 \text{ kg/s}$ find f .

$$\omega' = 2\pi f' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \Rightarrow f' = \frac{1}{2\pi} \sqrt{\frac{2.50}{0.300} - \frac{(0.90)^2}{4(0.30)^2}}$$

$$f' = 0.393 \text{ Hz}$$

b.) $b = ?$ for critically damped.

$$b = 2\sqrt{km}$$

$$b = 2\sqrt{(2.50 \text{ N/m})(0.300)} = 1.73 \text{ kg/s}$$

$$b = 1.73 \text{ kg/s}$$