Millikan Oil Drop Experiment

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The purpose of this experiment is to use Millikan's oil drop apparatus to observe the quantization of the electron *charge*, and to measure the fundamental charge of the electron $e = 1.602 \times 10^{-19}\ C$. Two methods are described in the Instruction Sheet on my website. The first method measures the charge on the oil droplet by suspending it in an electric field. The second method measures the charge on the same droplet by forcing it upward, using an electric field, and measuring its terminal velocity v_2. The charges are measured multiple times using both techniques to determine an average value for the charge on the droplet. This sequence of measurements is applied to multiple drops (e.g., 30 oil droplets) to (1) seek evidence for charge quantization, and (2) obtain the value of the fundamental charge e.

I. BACKGROUND

Robert Andrews Millikan constructed an experiment where oil was atomized (sprayed through a fine nozzle) to form tiny droplets between two oppositely charged parallel metal plates. Some of the oil droplets pick up one or more excess electrons while passing through the atomizer. Electrons are often added to the oil droplets due to friction with the atomizer nozzle as they are sprayed. With the application of an electric field E, the motion of the charged droplets can either (1) remain motionless due to an applied E-field, or (2) move upward at terminal velocity with the application of a larger E-field. Finally, the oil droplets can move downward at terminal velocity without an E-field being applied.

When Robert Millikan made this historic measurement (1909), scientists had already measured the charge-to-mass ratio (e/m) of the electron. While the e/m ratio was important for steering electron beams ($a = eE/m$), it did not reveal either the *mass* or the *charge* of the electron. The observation of charge quantization and its fundamental value (e) led scientists to a new discovery, namely, electric charge appeared in fundamental units of charge (i.e., $0e$, $\pm e$, $\pm 2e, \ldots$) and this discrete nature is found in all elementary particles (e.g., protons, neutrons, electrons, π^+, π^-, ...). Moving forward into the future (1960-1970), it was discovered that quarks and antiquarks ($q, \bar{q}$) have fractional units of charge ($\pm\frac{1}{3}e$, and $\pm\frac{2}{3}e$).

II. THEORY

In this application of the Millikan oil drop experiment, students will observe the motion of oil droplets due to a combination of forces applied to each oil droplet. **Take note**–the charge Q on each droplet can be different because more than one electron ($q = -e$) may be added to the oil droplet during the atomizing process. Students are expected to measure the charge (Q) of $\sim$30 oil droplets to acquire sufficient statistics. The radius of each droplet must be determined before a final correction can be applied to each of the measured charges ($Q \rightarrow Q_c$). The sequence of measurements are shown in Fig. 1. The measurements performed on each oil droplet include the following: (1) a **droplet in free-fall** to determine its radius, and (2) a **droplet in suspensiom** where the first value of Q is measured, and (3) a **droplet forced upward** were the second value of Q is measured.

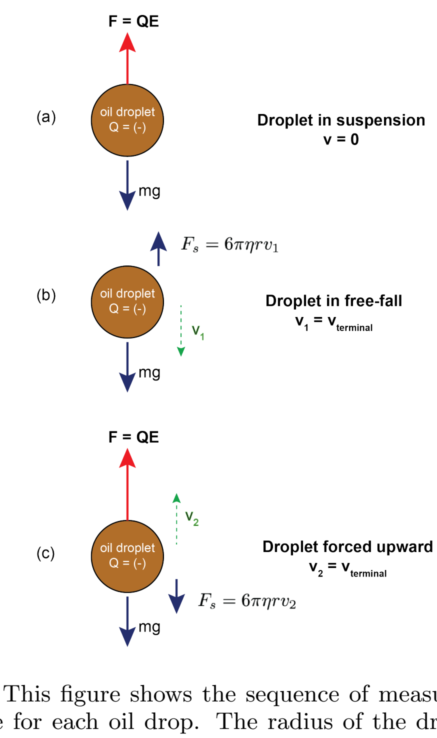

FIG. 1. This figure shows the sequence of measurements to complete for each oil drop. The radius of the droplet is determined from method (b). The value of Q is determined by using methods (b) and (c). If necessary, a second value of Q can be determined using methods (a) and (b). The electric field in (c) is obviously stronger than the electric field in (a)

A. Measurements

First of all, not all droplets observed in the Millikan oil drop experiment are the same. The radius and charge of each drop will vary, as you will see in your measurements. It is important that you measure the same oil droplet in the measurements that follow. The first measurement, the free-fall measurement (Fig. 1b), will determine the radius of the droplet. This measurement is used in both Methods I and II. If you choose Method I, the second measurement requires that you suspend the oil droplet in a motionless manner (Fig. 1a) to determine your measurement of Q. If you choose Method II, the second measurement requires that you force the oil droplet upwards (Fig. 1c) to determine your measurement of Q. These two measurements of Q should be in close agreement for the same droplet. You are encouraged to use Method II, so proceed with the measurements shown in Figs. 1b and 1c.

1. The oil droplet in free-fall

To determine the radius r of each oil droplet, we will use the *free-fall* technique (Fig. 1b) where we measure the terminal velocity (v_1) of each oil droplet. When the oil droplet is in free-fall, it reaches terminal velocity rather quickly due to the air-resistance which is determined, in part, by the mean-free-path of air molecules at STP which is typically $1/10^{\text{th}}$ the size of an oil droplet. The force due to the air resistance on a spherical object is called the Stoke's force ($F_{\text{S}} = 6\pi\eta r v$), where η is the viscosity of the air as the oil drop descends at terminal velocity, r is the radius of the droplet, and v is the *terminal velocity* in our case. The motion of the oil droplet is described by Newton's 1^{st} law:

$$- 6\pi\eta r v \, + \, mg \, = \, 0 \ . \tag{1}$$

where the +(pos) direction is chosen to be downward.

Take note: as the droplets *fall* under the influence of gravity, they appear as *rising* points of light when observed through the eyepiece. The eyepiece reverses the true motion of the oil droplets. Upward moving droplets seen through the eyepiece are really falling in the gravitational field, and *vice versa*. Expanding on Eq. 1, we can calculate the radius of the oil drop.

$$mg - 6\pi r v_1 \eta = 0$$

$$V\rho g - 6\pi r v_1 \eta = 0$$

$$\frac{4}{3}\pi r^3 \rho g \, - \, 6\pi r v_1 \eta = 0$$

$$r = \sqrt{\frac{9\eta v_1}{2\rho g}} \tag{2}$$

2. The stationary measurement – Method I

Now that the oil droplet radius is measured, we can proceed with either of the two measurements for Q as described in Figs. 4a (Method I) or 4c (Method 2).

Using the droplet radius determined by Eq. 2, the charge can be calculated by using the technique shown in Fig. 4a:

$$Q = \frac{6\pi d \eta v_1}{U_1}\sqrt{\frac{9\eta v_1}{2\rho g}} \tag{3}$$

Where

$$\eta = 1.81 \times 10^{-5} \ \left[\frac{Ns}{m^2}\right]$$

$$d = 6 \times 10^{-3} \ m$$

$$\rho_{oil} = 875.3 \ \frac{kg}{m^3}$$

$$\rho_{\text{air}} = 1.29 \ \frac{kg}{m^3}$$

$$\rho = 874 \ \frac{kg}{m^3}$$

$$g = 9.795 \ \frac{m}{s^2}$$

This yields a final equation of

$$Q = 2 \times 10^{-10} \, \frac{v_1^{\frac{3}{2}}}{U} \ \text{coulombs} \tag{4}$$

Once you have the local air pressure (see below–Section VI), you can make a correction to the density of air. The density of air (ρ_{air}) at our altitude is approximately 80% of what is quoted in the number above.

B. Method 2–We will use this technique

The second method requires making two measurements, the free fall velocity (v_1) of the droplet (Fig. 1b) and measuring the upward terminal velocity (v_2) for the droplet under the influence of an electric field as shown in Fig. 1c.

Using both techniques, the charge Q can be calculated using the following equation:

$$Q = (v_1 + v_2)\frac{\sqrt{v_1}}{U_2}\eta^{3/2}\frac{18\pi d}{\sqrt{2\rho g}} \tag{5}$$

where v_2 is the *rising* terminal velocity of the oil droplet, and d is the separation between the plates. This equation can be simplified using the same constants as shown in Method I:

$$Q = (v_1 + v_2)\frac{\sqrt{v_1}}{U_2}(2 \times 10^{-10}C) \tag{6}$$

C. Correction Factor

No matter which method you use, the temperature and air pressure affect different sized oil droplets. As a result, a correction factor must be introduced to include the temperature and pressure. The corrected charge Q_c is given by:

$$Q_c = \frac{Q}{(1 + \frac{b}{rP})^{3/2}} \tag{7}$$

where P is the pressure, r is the radius of the droplet, and b is a constant. The reason for including this correction is due to the size of the oil droplet. The size of the droplets are so small that the motion of the air molecules begin to influence their motion when undergoing *free-fall*. You can imagine that if the oil droplet becomes very small, it will become suspended in the air (e.g., like small drops of water in foggy weather).

III. OBJECTIVE

Once again, the purpose of this lab is to observe the *quantization of charge* and determine the *size* of the fundamental charge, e.

IV. EQUIPMENT

The Millikan oil-drop apparatus is shown in Fig. 2. A fine spray of oil is injected in the region between the horizontal capacitor plates that are connected to an external power supply. The motion of the oil drops are observed using an eyepiece with graduated cross-hairs. The operation of the oil-drop apparatus is described in the operations manual for this experiment.

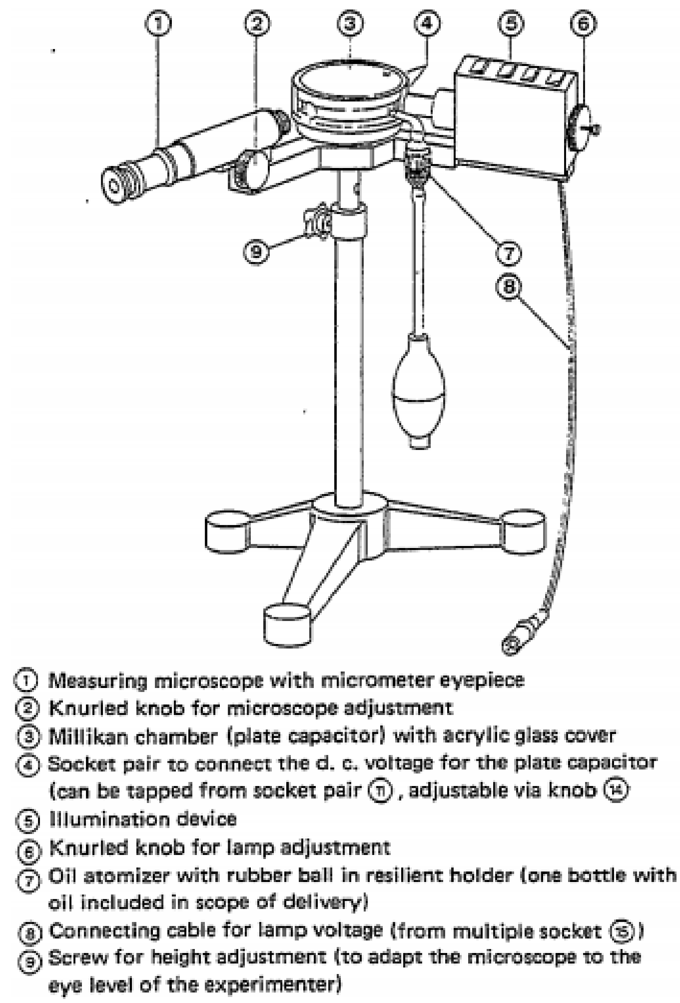

FIG. 2. Millikan Apparatus

V. DATA ACQUISITION

The physical measurements collected in this experiment include the voltage U, the free-fall distance S_1, and the free-fall time t_1. Make sure to include the uncertainties of these measurements in your log book. Make multiple measurements for each droplet and gather data for ~ 30 droplets. Leave columns in your table to calculate the radius, the charge Q, and the corrected charge Q_C for each droplet.

Take care while recording the free-fall distance S_1. When looking through the eyepiece, the separation between minor ticks is $100\,\mu$ while the separation between major ticks is 1 mm. However, these measurements are magnified by a factor (1.875), so, a correction needs to be made for the vertical distances recorded. Most likely, you will record your distance using the number of *minor ticks*.

The corrected distance is calculated using the following equation:

$$S_1 = Ticks \times \frac{10^{-4}}{1.875}\ [m]$$

where S_1 is the corrected distance, and *Ticks* refers to the number of *minor* tick marks over which the oil droplet fell. The free-fall velocity is calculated using the following equation:

$$v_1 = \frac{S_1}{t_1}\left[\frac{\text{m}}{\text{s}}\right]$$

where v_1 is the free-fall velocity and t_1 is the free-fall time. A similar equation is used for oil droplets rising in an electric field (Fig. 1c).

Use your measurements of v_1 and v_2 in Eq. 6 to calculate Q using Method II.

A. Method 2

Again, we are not going to pursue Method I in our analysis of the Millikan oil-drop experiment. So, you only need to use the procedures described in Fig. 1b and 1c to calculate Q, r, and Q_c.

B. Questions for this lab

You must answer the following questions in your logbook as part of this lab.

1. What is the typical density of air in our lab on the day you took your measurements? (kg/m^3).
2. What is the mathematical expression for the uncertainty of $(v_1 + v_2)\sqrt{v_1}$ shown in Eq. 6?
3. What is the accepted value of e? (______ ± ______) coulombs
4. How far must the droplet fall, starting from rest, before it reaches terminal velocity? (μm) You might want to refer to chapter 5 in your University Physics textbook to review the material on terminal velocity.

C. Correction Factor

As mentioned before, the charge Q must be corrected to take into account the radius of each droplet. Based on Eq. 7, the corrected charge Q_C can be calculated. The constant b must be determined graphically and this is described in the Instruction Leaflet (found on my website).

VI. SPECIAL CONSIDERATIONS

1. There is high voltage in this experiment, so be aware of this.

2. Use a bubble level twice (in perpendicular directions) to make sure the Millikan Oil-Drop chamber is level before making measurements.

3. You will probably want to drape a dark cloth over your head and the experiment as you make the measurements. The experiment has its own light source to light up the oil drops as you view them in the apparatus

4. You can find the current "Surface Air Pressure" on our meteorology website. Go down the right side of the web page and "click" on *Current AC 1 rooftop weather*. You will find the surface air pressure in units of hPa (hectoPascals). If the ERAU Meteorology site is not functioning properly, try using the Phyphox app on your cellphone to obtain the current air pressure.