Cavendish Experiment – Measuring G

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The purpose of this experiment is to measure the universal gravitational constant, G, a constant that describes the strength of the gravitational force in Nature. This experiment measures the torque produced in a torsion pendulum due to the inverse r^2 gravitational force first described by Isaac Newton. Even though gravitational forces produced in this experiment are very weak, it is nonetheless sensitive enough to measure them and give an approximate value of G.

I. BACKGROUND

The computerized Cavendish balance is a torsion pendulum whose angular motion is recorded on a computer. The torsion pendulum is suspended by a tungsten wire designed to give a period of oscillation on the order of $3\frac{1}{2}$ to $4\frac{1}{2}$ minutes. The boom supporting the two "small" lead balls on the wire balance is an aluminum plate that changes the capacitance, and thus the voltage, read out by the electronics. This is a rather clever approach because it leads to minimal setup time, and provides a reliable technique for measuring the angular deflection.

The two large lead balls shown in Fig. 1 are located on an external boom that can be moved from one extreme position to the other. As you might suspect, the large lead balls attract the smaller lead balls suspended on the torsion pendulum, and their mutual attraction can be measured by observing the rotation of the torsion pendulum.

The Cavendish balance is one of the most sensitive devices for measuring the force between two objects. It has been used to measure the strengths of both the gravitational and electrostatic forces. You will discover during the course of your measurements how sensitive this device is. For your information, we have a very sensitive Cavendish balance on campus that measures down to 1.0 pico-newtons.

II. THE EXPERIMENT

In this experiment, the force between the two lead balls can be described by Newton's universal law of gravitation:

$$F = -G\,\frac{M\,m}{r^2} \tag{1}$$

where M is the mass of the large lead ball, m is the mass of the small lead ball, and r is the distance between their respective centers of mass. The universal gravitational constant G is what you are attempting to measure. The accepted value for G is 6.67×10^{-11} N·m^2/kg^2.

The force described by Eq. 1 produces torques that cause the torsion pendulum to undergo angular acceleration. When the large masses are placed at one extreme position, the torsion pendulum undergoes oscillations until it reaches its "new" equilibrium position. Finding the equilibrium angles for both extreme positions will be an important step in the data you collect. Once the torsion pendulum is in its equilibrium position, the external torque can be described by the following equation:

$$\tau \;=\; -\,k\,\theta_{\rm max} \tag{2}$$

where $\theta_{\rm max}$ is the displacement from the "no torque" equilibrium position. The torsion constant k is a property of the tungsten wire supporting the balance, and is proportional to the *stiffness* of the wire, while τ is the external torque causing the angular displacement from the "no torque" equilibrium position.

III. THE EQUIPMENT

The Cavendish balance is thoroughly described in the supporting literature found on my website: `physicsx.pr.erau.edu`. If you have been assigned to use this equipment, chances are that the professor and/or the lab assistant have already done much of the preliminary work required to make the balance work. The most delicate part of this experiment is the tungsten wire. While tungsten is a strong material, it is also brittle. If for some reason you need to move the Cavendish balance any appreciable distance, please ask your instructor for assistance. A new stop was design and implemented to facilitate transporting the Cavendish balance–thank you Dr. A. Gretarsson.

IV. PROCEDURE

A. Download Software from TelAtomic

You will need to download the software from the Tel Atomic website to perform real-time data acquisition.

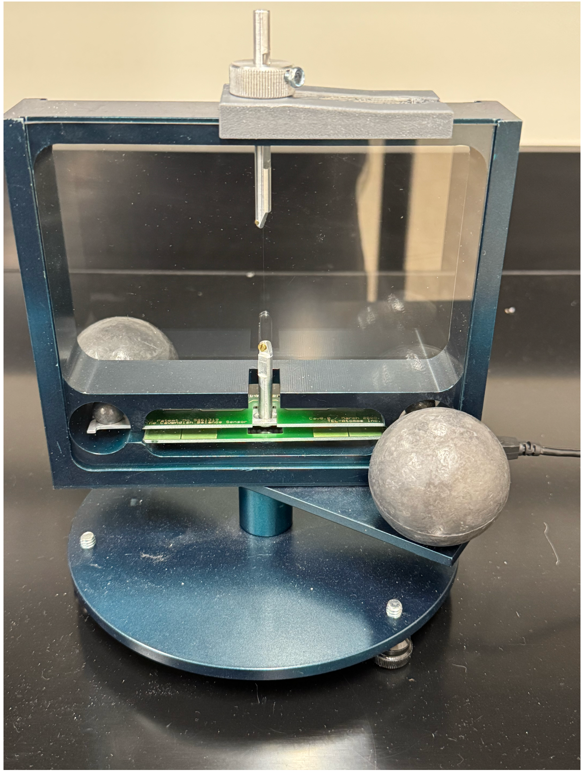

FIG. 1. The Cavendish balance used to measure G is shown in the figure above. Two large lead balls can be observed on opposite sides outside the torsion balance box. Inside the box are two smaller lead balls that are torqued due to the gravitational influence of the two large lead balls outside the pendulum apparatus. The pendulum is supported by a thin 20μ tungsten wire.

B. Additional Resources

You will find more information regarding the experimental procedure in the following material:

1. my `physicsx` webpage,
2. your University Physics textbook–damped harmonic motion in chapter 14.

You will find two leaflets on my `physics` website describing the apparatus and some of the equations used in the calculations, in particular, the equation for damped harmonic motion:

$$\theta_+(t) = ae^{-bt}\cos(\omega_d t + \phi) + \theta_+ \tag{3}$$

where five parameters (a, b, ω_d, ϕ, and θ_+) must be fit to Eq. 3. The above equation is used to describe the damped harmonic motion when the large lead balls are in the extreme clockwise position (causing the small lead balls to move in $+\theta$). A similar equation using θ_- describes the damped harmonic motion when the large lead balls are in the extreme counter-clockwise position (causing the small lead balls to move in -θ). The fitted constants θ_+ and θ_- describe where the torsion pendulum would eventually come to rest if you wait 5-6 hours. However, you don't have time for that, so, this is a short-cut to getting those results.

Recall that the angular frequency for a damped harmonic oscillator has the form:

$$\omega_d^2 = \omega_0^2 - b^2 \tag{4}$$

where ω_d and b (the damping parameter) are what you measure from the fit. The value ω_0 is the natural oscillation frequency if there were no energy dissipation (i.e., $b \approx 0$). Finally, the torsion constant K can be found from the natural oscillation frequency ω_0. From chapter 14 in University Physics, one finds for a torsion pendulum that $\omega_0^2 = K/I$ where I is the moment of inertia of the small-mass platform. This leads to a K value:

$$K = \left(\omega_d^2 + b^2\right) I \tag{5}$$

Using this value of the torsion constant, one can proceed with calculating G. A coarse calculation is shown in Eq. 18 in the "Computerized Cavendish Balance" writeup, while a more precise calculation is shown in Eq. 34. In either case, you need the torsion constant K.

C. Cavendish Output

The data from your Cavendish experiment should be saved in `.csv` format for post-processing later on. Your data will track better to damped harmonic motion if the amplitude has a swing of more than 8-10 mrad from peak-to-trough. If the initial amplitude is < 5 mrad from peak-to-trough, you will find that the data do not track well with a damped harmonic fit as shown in Fig. 2.

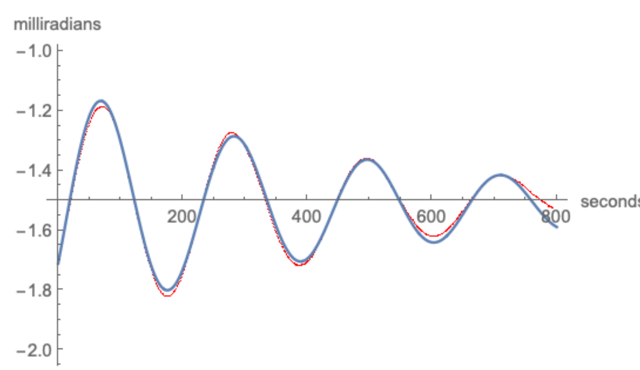

FIG. 2. This plot shows the data (red) and the best fit curve (blue) using Eq. 3. Low amplitude oscillations, such as the one shown here, lead to poor tracking between the data and the fitted function.

D. Classical Physics Writeup

This leaflet describes the apparatus in its entirety. Focus your attention on pages 4-6 as you begin setting up the experiment. In most cases the calibration has been done. For the time being, assume it works unless the entire range of angular motion cannot be recorded (i.e., calibration is needed).

Starting on page 10 of this writeup is "Driven Resonance Method" for measuring G. Skip it, and go to page 16 and use the "Static Method" for measuring G. Take note that there are a long list of corrections that one must include to improve the accuracy of G and this goes all the way to page 21.

Appendix 1, starting on page 22, describes the Cavendish Balance Software. It's worth your time reviewing it, and it's a *quick read*. The software can be found at Tel-Atomic.

E. Supplemental Material

This leaflet is a one-pager. The equations should be familiar to you from your Physics II studies of simple harmonic motion, and damped harmonic motion. Recall that the damping term b affects the measured oscillation frequency $\omega_d = 2\pi/T$.

N.B. Please answer the questions in paragraph 4 of this leaflet (i.e., the one-pager), and proceed on with the instructions outlined in paragraph 5.

V. IMPORTANT CONSIDERATIONS

- Balance the Cavendish platform (using a level) in at least two directions before taking any data.
- Don't forget to calibrate your balance by recording the extreme angular displacements ($\pm$70.0 mrad)–if it hasn't already been done.
- **Be gentle with this apparatus!** The tungsten wire can be easily broken.]
- Do not move the Cavendish balance before checking with the lab assistant or the instructor.
- The Cavendish balance is the only sensitive piece of equipment in this experiment.
- There is no high voltage or other health hazards associated with this experiment, unless you drop the lead ball on your foot. Don't do that !!
- Make sure to follow the procedures describing the list of corrections to improve the accuracy of your measurement of G.
- **Make sure** you include a thorough writeup of your error analysis. Make sure you answer the questions in paragraph 4 of the "Supplemental Material."

VI. USEFUL MEASUREMENTS FOR THIS LAB

small Pb sphere
$m = 0.014559 \pm 0.000001$ kg
$r = 0.006730 \pm 0.000048$ m

large Pb sphere
$M = 1.038550 \pm 0.000707$ kg
$R = 0.0281025 \pm 0.000013$ m

Al Beam
$m_{\text{beam}} 0.007174 \pm 0.000010$ kg
$\ell_{\text{beam}} = 0.148000 \pm 0.001$ m
$w_{\text{beam}} = 0.012730 \pm 0.00030$ m
$2r_{\text{hole-to-hole}} = 0.133 \pm 0.001$ m

Separation between the glass slides

separation $= 0.035100 \pm 0.00010$ m *inner-to-outer*