Chapter 17
Temperature and heat

1 Temperature and Thermal Equilibrium

When we speak of objects being "hot" and "cold", we need to quantify this by some scientific method that is quantifiable and reproducible. Before we introduce the concept of temperature, we must first establish the concept of *thermal equilibrium.*

When two systems are placed in contact through a diathermic wall, the passage of heat energy through the wall–if it occurs–causes the properties of the two systems to change. When all the measured properties of each system approach constant values, we say the two systems are in *thermal equilibrium*

> *If systems A and B are each in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.*

This is called the *zeroth law of thermodaynamics*.

When two systems are in thermal equilibrium, we say that they have the same *temperature.* System C essentially plays the role of the thermometer. Another way of stating the **zeroth law in terms of temperature** is:

> *There exists a scalar quantity called temperature, which is a property of all thermodynamic systems in equilibrium. Two systems are in thermal equilibrium if and only if their temperatures are equal.*

2 Thermometers and Temperature Scales

Historically, there were two temperature scales (Fahrenheit and Celsius) that were produced as a practical convenience and neither of them have a deep physical meaning. The scale that is universally adopted as fundamental in physics is the *Kelvin scale* where "absolute zero" is defined as zero on the Kelvin scale. The increments in the Kevlin scale are called degrees and identically match the separation in Celsius degrees.

In order to calibrate thermometers between laboratories, there needs to be a specific temperature that can be reliably reproduced. One of these temperatures is called the *triple-point of water*, the temperature where water, ice, and water vapor coexist at atmospheric pressure. In the $P-T$ diagram, the *vapor pressure* of water at 0.01 ^{o}C is 610 Pa. The triple-point of water is defined to be $T_{\rm tr} = 273.16$ K (exactly).

The Celsius and the Fahrenheit Temperature Scales

The Celsius scale was developed around the boiling point and freezing point of water ($\rm T_{boil} = 100^oC$ and $\rm T_{freezing} = 0^oC$). The relationship between the Celsius scale and the Kelvin scale is:

$$T_C \;=\; T - 273.15$$

The Fahrenheit scale was developed around several choices but later came to be fixed around the boiling point and freezing point of water ($\rm T_{boil} = 212^oF$ and $\rm T_{freezing} = 32^oF$). The relationship between the Celsius scale and the Fahrenheit scale is:

$$T_F \;=\; \frac{9}{5}\,T_C + 32$$

where the intervals between the two temperature scales is 9 $\rm F^o = 5\ C^o$.

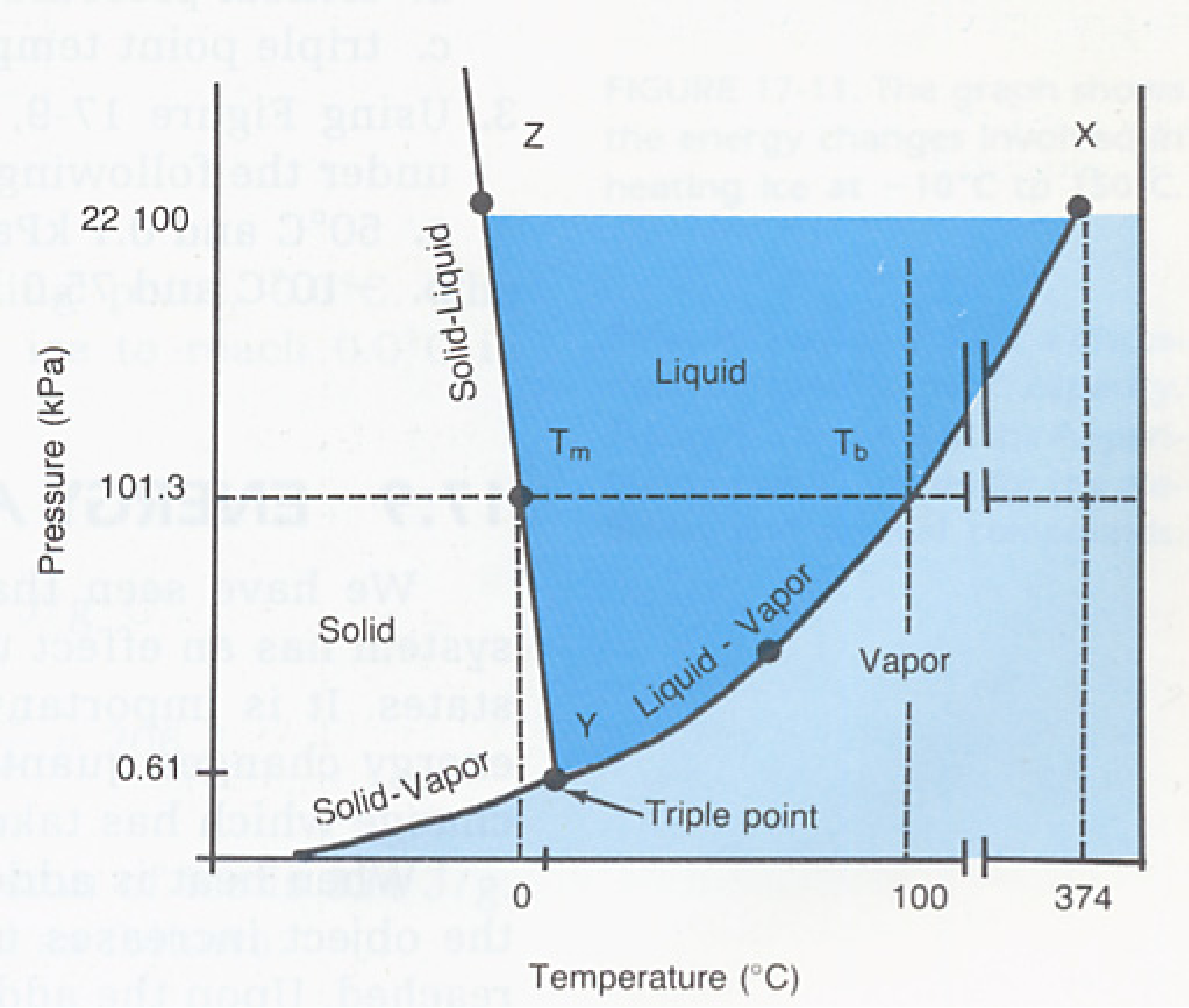


Figure 1: This figure shows the vapor pressure of water at its triple-point temperature (0.01^{o}C), at 100^{o}C, and at its critical-point temperature (374^{o}C).

Ex. 8 Convert the following Kelvin temperatures to the Celsius and Fahrenheit scales: (a) the midday temperature at the surface of the moon (400 K); (b) the temperature at the tops of the clouds in the atmospere of Saturn (95 K); (c) the temperature at the center of the sun ($1.55{\times}10^7$ K).

3 Gas Thermometers and the Kelvin Scale

In principle, any property (X) of a substance that varies with temperature can form the basis for a thermometer. Examples might include the volume of a liquid, the pressure of a gas kept at constant volume, the electrical resistance of a wire, etc. In general, we can calibrate any device to the triple-point temperature of water.

$$T(X) \;=\; (273.16\ \mathrm{K})\,\frac{X}{X_{\mathrm{tr}}}$$

where X is the pressure, volume or electrical resistance (whatever) that is changing as a function of temperature.

The thermometric property that proves most suitable for measuring temperatures on the Kelvin scale is the pressure p exerted by a fixed volume of gas. Such a device is called a *constant-volume gas thermometer*.

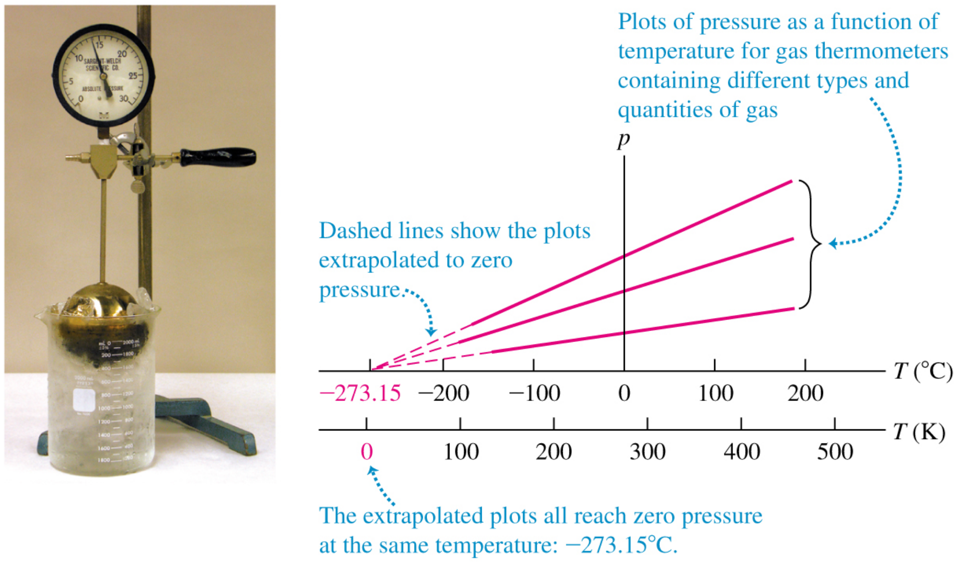
If we plot these points on a T-P diagram, we see that the slope intersects the temperature (T) axis at $p = 0$. The temperature at this point is regarded as the temperature of the system and we define it as the *ideal gas temperature scale*:

$$T \;=\; (273.16\ K)\,\frac{p}{p_{\mathrm{tr}}} \qquad \text{(constant V)} \tag{1}$$

Ex. 7 The pressure of a gas at the triple point temperature of water is 1.35 atm. If its volume remains unchanged, what will its pressure be at the temperature CO_2 solidifies $T_{\mathrm{CO}_2} = 195K$?

(a) A constant-volume gas thermometer

(b) Graphs of pressure versus temperature at constant volume for three different types and quantities of gas



Plots of pressure as a function of temperature for gas thermometers containing different types and quantities of gas

Dashed lines show the plots extrapolated to zero pressure.

The extrapolated plots all reach zero pressure at the same temperature: −273.15°C.

4 Thermal Expansion

Different materials expand to different lengths for the same temperature difference ΔT. The change in length ΔL resulting from a change in temperature ΔT can be written as:

$$\Delta L \;=\; \alpha\, L\, \Delta T \qquad \text{(linear thermal expansion)}$$

where α is called the *coefficient of linear expansion.* The coefficient α is defined as the fractional change in length per unit *temperature difference*, or

$$\alpha \;=\; \frac{\Delta L / L}{\Delta T} \qquad\qquad [\alpha] = (\mathrm{C^o})^{-1}$$

In two dimensions, we have:

$$\Delta A = 2\alpha\, A\, \Delta T \qquad \text{(for an isotropic solid)}$$

while in three dimensions, we have:

$$\Delta V \;=\; 3\alpha\, V\, \Delta T \qquad \text{(for an isotropic solid)}$$

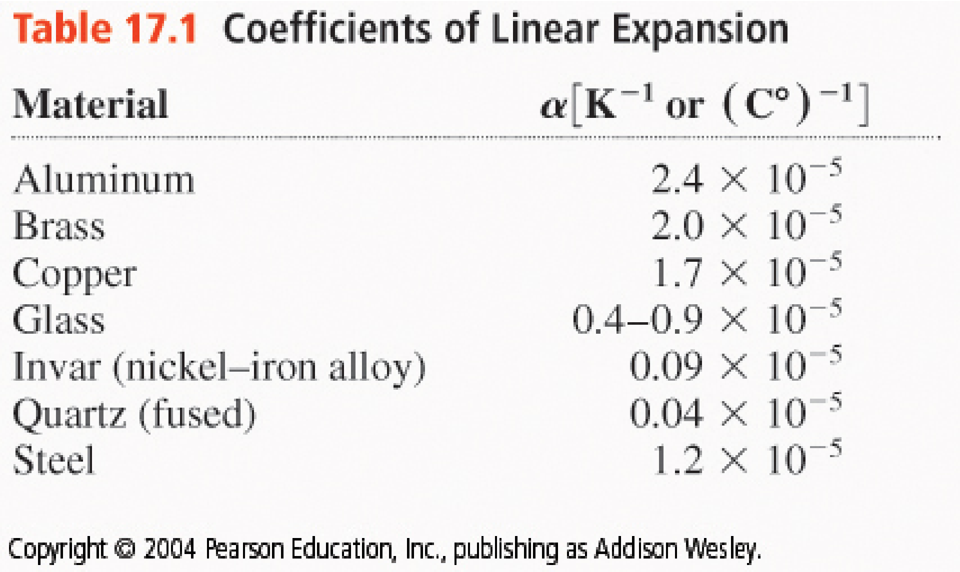


Table 17.1 Coefficients of Linear Expansion

Material	$\alpha\,[\mathrm{K}^{-1}$ or $(\mathrm{C}^{\circ})^{-1}]$
Aluminum	2.4×10^{-5}
Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4–0.9 \times 10^{-5}$
Invar (nickel–iron alloy)	0.09×10^{-5}
Quartz (fused)	0.04×10^{-5}
Steel	1.2×10^{-5}

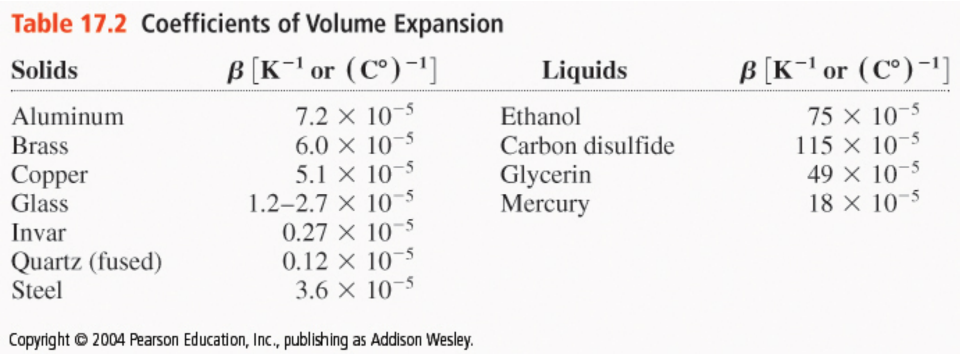


Table 17.2 Coefficients of Volume Expansion

Solids	$\beta\,[\mathrm{K}^{-1}$ or $(\mathrm{C}^{\circ})^{-1}]$	Liquids	$\beta\,[\mathrm{K}^{-1}$ or $(\mathrm{C}^{\circ})^{-1}]$
Aluminum	7.2×10^{-5}	Ethanol	75×10^{-5}
Brass	6.0×10^{-5}	Carbon disulfide	115×10^{-5}
Copper	5.1×10^{-5}	Glycerin	49×10^{-5}
Glass	$1.2–2.7 \times 10^{-5}$	Mercury	18×10^{-5}
Invar	0.27×10^{-5}		
Quartz (fused)	0.12×10^{-5}		
Steel	3.6×10^{-5}		

Ex. 14 **Ensuring a Tight Fit.** Aluminum rivets used in airplane construction are made slightly larger than the rivet holes and cooled by "dry ice" (solid CO_2) before being driven. If the diameter of a hole is 4.500 mm, what should be the diameter of a rivet at 23.0°C, if its diameter is to equal that of the hole when the rivet is cooled to -78.0°C, the temperature of dry ice? Assume that the expansion coefficient remains constant at the value given in Table 17.1.

The above equations cannot be used for fluids. Instead, we define the *coefficient of volume expansion* β of a fluid by the following equation:

$$\Delta V \;=\; \beta V \, \Delta T \qquad \text{(for liquids)}$$

Most liquids expand with increasing temperature (i.e., $\beta > 0$). However, take a look at water around 4^oC. In gases, β is strongly dependent upon temperature; in fact, for an ideal gas $\beta = 1/T$ where T is expressed in kelvins.

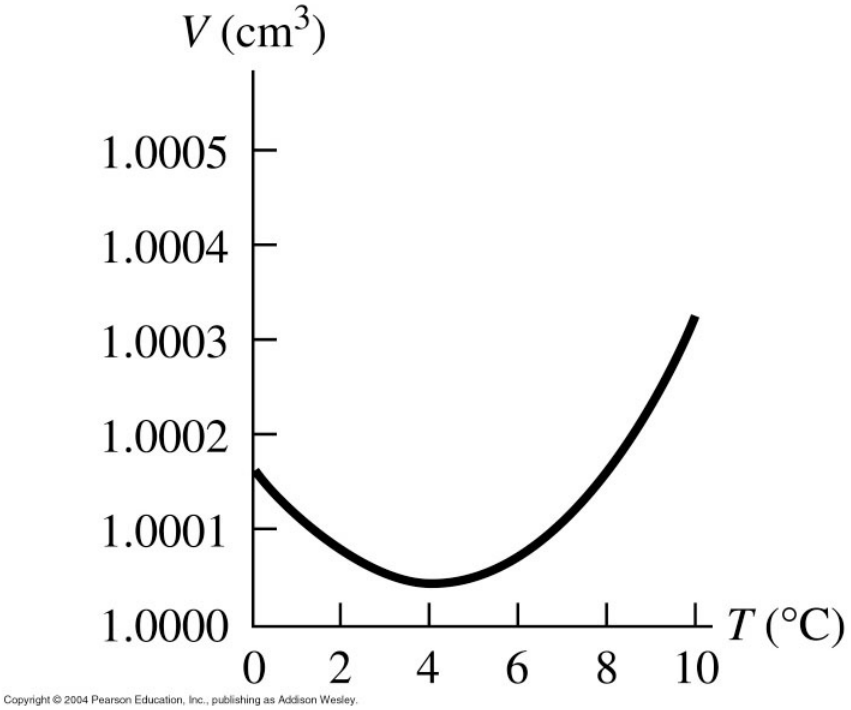
Let's define the area $A = ab$ and the final area after the thermal expansion $A' = (a + \Delta a)(b + \Delta b)$. Then $\Delta A = A' - A$,

$$\Delta A = a\Delta b + b\Delta a + \Delta a \, \Delta b$$

Then the fractional change in area becomes:

$$\frac{\Delta A}{A} \;=\; \frac{\Delta a}{a} + \frac{\Delta b}{b} + \underbrace{\frac{\Delta a \, \Delta b}{a \, b}}_{\text{very small}} \;\approx\; \alpha \, \Delta T \,+\, \alpha \, \Delta T$$

4.1 Thermal expansion of water



4.2 Thermal Stress

In this section, we "connect" two concepts, the fractional change due to *temperature change*, and the fraction change due to *compression* or *tensile* forces.

$$\left(\frac{\Delta L}{L_o}\right)_{\text{thermal}} = \alpha\,\Delta T$$

and

$$\left(\frac{\Delta L}{L_o}\right)_{\text{tension}} = \frac{F}{AY}$$

Solving for the tensile stress (F/A) required to keep the rod's length constant, we find:

$$\frac{F}{A} = -Y\,\alpha\,\Delta T$$

Ex. 22 A brass rod is 185 cm long and 1.60 cm in diameter. What force must be applied to each end of the rod to prevent it from contracting when it is cooled from 120°C to 10°C? $(4 \times 10^4$ N).

5 Quantity of Heat

The heat capacity C of a body is the ratio of the amount of heat energy Q transferred to a body in any process to its corresponding temperature change ΔT.

$$C = \frac{Q}{\Delta T} \qquad \text{(heat capacity)}$$

The heat capacity per unit mass is called the *specific heat capacity* or just the *specific heat.*

$$c = \frac{C}{m} = \frac{Q}{m\,\Delta T} \qquad \text{(the specific heat)}$$

Units of specific heat

Specific Heat Capacity – J/(kg·K)

Molar Heat Capacity – J/(mol·K)

Here are some useful conversions:

1 cal = 4.186J
1 kcal = 1000 cal = 4186 J
1 Btu = 778 ft·lb = 252 cal = 1055 J

The heat Q required to raise the temperature of a substance by ΔT is:

$$Q = mc\,\Delta T$$

where m is the mass and c is the **specific heat** measured in *joules/(kg·K)*.

There is another equation describing heat flow that is often used when the amount of material is measured in *moles*, instead of kilograms. In this case

$$Q = nMc\,\Delta T = nC\,\Delta T$$

where C is the molar specific heat measured in *joules/(mol·K)*.

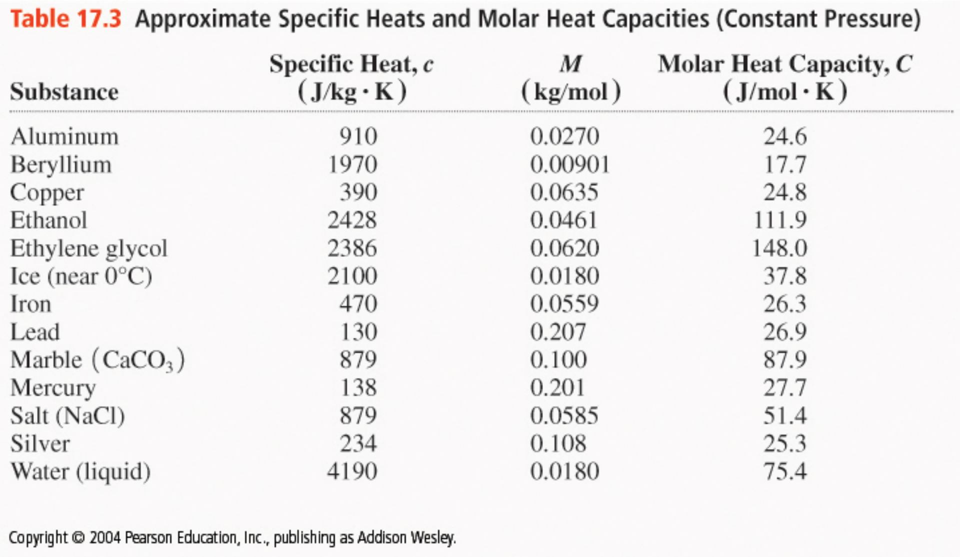


Table 17.3 Approximate Specific Heats and Molar Heat Capacities (Constant Pressure)

Substance	Specific Heat, c (J/kg · K)	M (kg/mol)	Molar Heat Capacity, C (J/mol · K)
Aluminum	910	0.0270	24.6
Beryllium	1970	0.00901	17.7
Copper	390	0.0635	24.8
Ethanol	2428	0.0461	111.9
Ethylene glycol	2386	0.0620	148.0
Ice (near 0°C)	2100	0.0180	37.8
Iron	470	0.0559	26.3
Lead	130	0.207	26.9
Marble ($CaCO_3$)	879	0.100	87.9
Mercury	138	0.201	27.7
Salt (NaCl)	879	0.0585	51.4
Silver	234	0.108	25.3
Water (liquid)	4190	0.0180	75.4

Ex. 31 While painting the top of an antenna 225 m in height, a worker accidentally lets a 1.00-L water bottle fall from his lunch-box. The bottle lands in some bushes at ground level and does not break. If a quantity of heat equal to the magnitude of the change in mechanical energy of the water goes into the water, what is its increase in temperature?

6 Calorimetry and Phase Changes

Method of Mixtures

When different materials of different temperatures are brought in thermal contact, they will exchange heat until they come to thermal equilibrium (i.e., they have the same temperature). For example if three materials are in thermal contact with each other, there will be an exchange of thermal energy (Q) until they attain the same temperature:

$$Q_1 + Q_2 + Q_3 = 0 \qquad \text{(conservation of energy)}$$

Ex. 35 A 500-g chunk of an unknown metal, which has been in boiling water for several minutes, is quickly dropped into an insulating Styrofoam beaker containing 1.00 kg of water at room temperature (20.0°C). After waiting and gently stirring for 5.00 minutes, you observe that the water's temperature has reach a constant value of 22.0°C. a) Assuming that the Styrofoam absorbs a negligibly small amount of heat and that no heat was lost to the surroundings, what is the specific heat of the metal? b) Which is more useful for storing heat, this metal or an equal weight of water? Explain. c) What if the heat absorbed by the Styrofoam actually is not negligible. How would the specific heat you calculated in part (a) be in error? Would it be too large, too small, or still correct? Explain

Heats of Transformation

When materials change from one *phase* to another (i.e., solid → liquid, or liquid → gas), heat enters (or leaves) the system, but the temperature doesn't change. This is called *latent heat* or *latent heat of transformation.* The total heat transferred in a phase change is:

$$Q = mL$$

where m is the mass and L_f is the latent heat of fusion (solid ↔ liquid), and L_v is the latent heat of vaporization (liquid ↔ gas).

Ex. 45 What must the initial speed of a lead bullet be at a temperature of 25°C so that the heat developed when it is brought to rest will be just sufficient to melt it? Assume that all the initial mechanical energy of the bullet is converted to heat and that no heat flows from the bullet to its surroundings. (Typical rifles have muzzle speeds that exceed the speed of sound in air, which is 347 m/s at 25°C.)

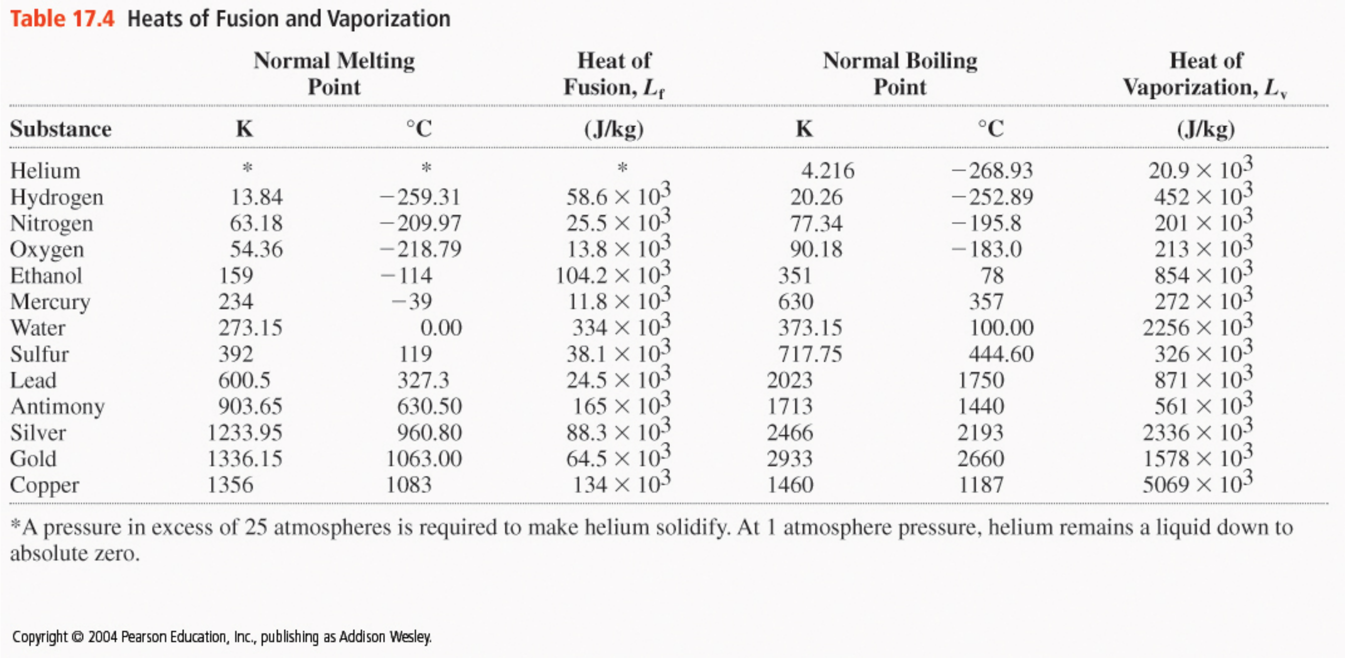
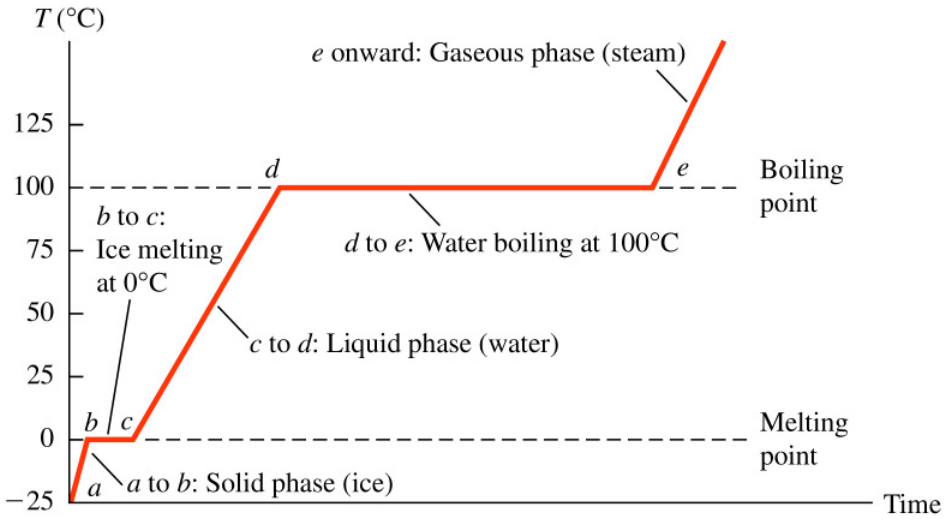


Table 17.4 Heats of Fusion and Vaporization

	Normal Melting Point		Heat of Fusion, L_f	Normal Boiling Point		Heat of Vaporization, L_v
Substance	**K**	**°C**	**(J/kg)**	**K**	**°C**	**(J/kg)**
Helium	*	*	*	4.216	−268.93	20.9×10^3
Hydrogen	13.84	−259.31	58.6×10^3	20.26	−252.89	452×10^3
Nitrogen	63.18	−209.97	25.5×10^3	77.34	−195.8	201×10^3
Oxygen	54.36	−218.79	13.8×10^3	90.18	−183.0	213×10^3
Ethanol	159	−114	104.2×10^3	351	78	854×10^3
Mercury	234	−39	11.8×10^3	630	357	272×10^3
Water	273.15	0.00	334×10^3	373.15	100.00	2256×10^3
Sulfur	392	119	38.1×10^3	717.75	444.60	326×10^3
Lead	600.5	327.3	24.5×10^3	2023	1750	871×10^3
Antimony	903.65	630.50	165×10^3	1713	1440	561×10^3
Silver	1233.95	960.80	88.3×10^3	2466	2193	2336×10^3
Gold	1336.15	1063.00	64.5×10^3	2933	2660	1578×10^3
Copper	1356	1083	134×10^3	1460	1187	5069×10^3

*A pressure in excess of 25 atmospheres is required to make helium solidify. At 1 atmosphere pressure, helium remains a liquid down to absolute zero.



Ex. 52 A 4.00-kg silver ingot is taken from a furnace, where it temperature is 750°C, and placed on a large block of ice at 0°C. Assuming that all the heat given up by the silver is used to melt the ice, how much ice is melted?

7 Mechanisms of Heat Transfer

There are three mechanisms by which heat transfer takes place:

1. conduction
2. convection, and
3. radiation

Thermal Conduction
The rate of heat transfer [J/s] through a thin slab of homogeneous material of thicknes Δx and area A with one face held at a constant temperature T and the other at a somewhat higher constant temperature $T + \Delta T$ is:

$$H = \frac{Q}{\Delta t} = kA\frac{\Delta T}{\Delta x} \tag{2}$$

where k is called the *thermal conductivity* of the material, and $\Delta x = L$, the length of the rod. The SI units of k is (W/m·K).

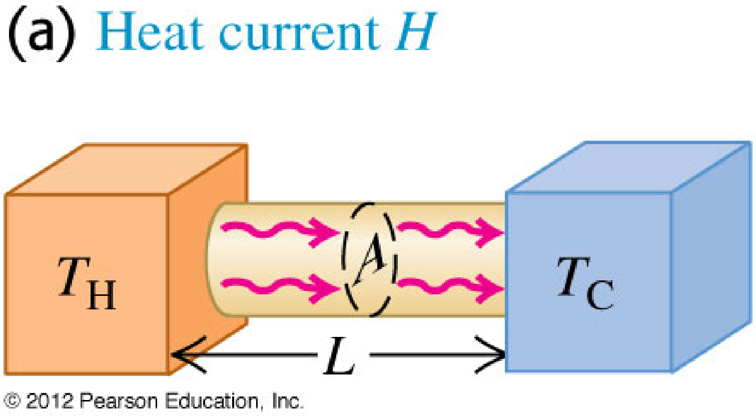


Figure 2: Figure 17.24 from University Physics 15th edition.

Look at the table of thermal conductivities. The *thermal resistance* or R-value is defined by $R = L/k$ where L is the thickness of the material through which the heat is transferred.

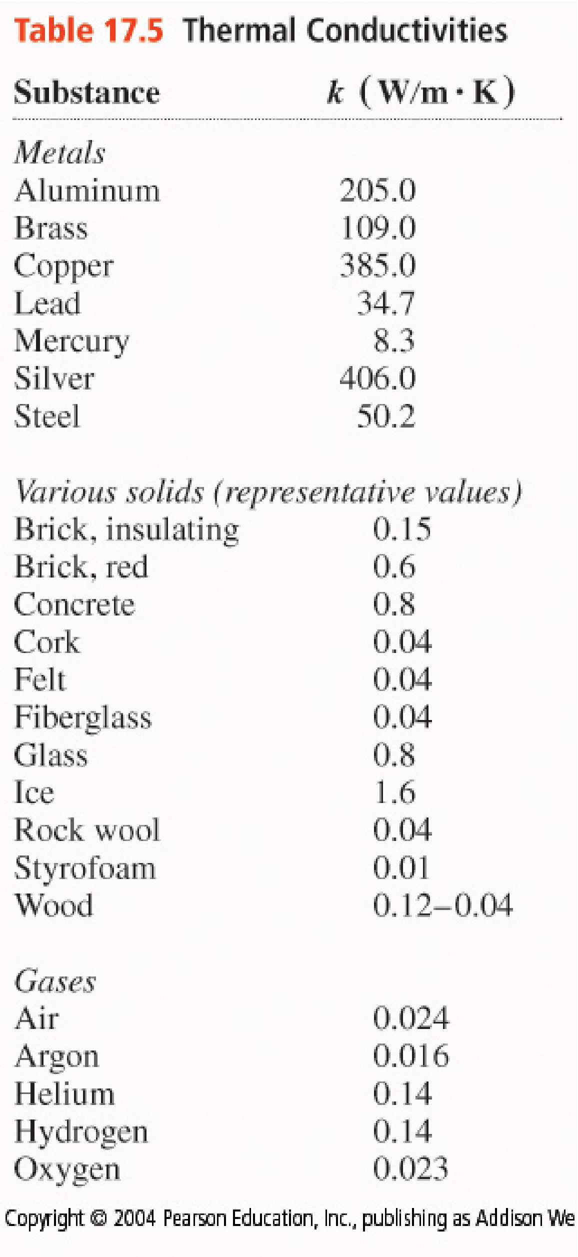


Table 17.5 Thermal Conductivities

Substance	k (W/m·K)
Metals	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2
Various solids (representative values)	
Brick, insulating	0.15
Brick, red	0.6
Concrete	0.8
Cork	0.04
Felt	0.04
Fiberglass	0.04
Glass	0.8
Ice	1.6
Rock wool	0.04
Styrofoam	0.01
Wood	0.12–0.04
Gases	
Air	0.024
Argon	0.016
Helium	0.14
Hydrogen	0.14
Oxygen	0.023

Figure 3: Thermal Conductivities from University Physics 13th edition.

There are two applications for the above equation.

$$H \;=\; kA\left(\frac{T_H - T_L}{L}\right) \qquad \text{(macroscopic, steady-state systems)} \tag{3}$$

and

$$H \;=\; -kA\,\frac{dT}{dx} \qquad \text{(microscopic, steady-state systems)} \tag{4}$$

where dT/dx is called the *temperature gradient*. In Eq. 4 dT/dx is intrinsically negative because $+x$ is assumed to be the direction of heat flow H $(+)$.

Equation 4 is useful for "non-flat" geometries such as cylindrical and spherical geometries.

Ex. 56. One end of an insulated metal rod is maintained at 100°C, and the other end is maintained at 0°C by an ice-water mixture. The rod is 60.0 cm long and has a cross-sectional area of 1.25 cm^2. The heat conducted by the rod melts 8.50 g of ice in 10.0 min. Find the thermal conductivity k of the metal.

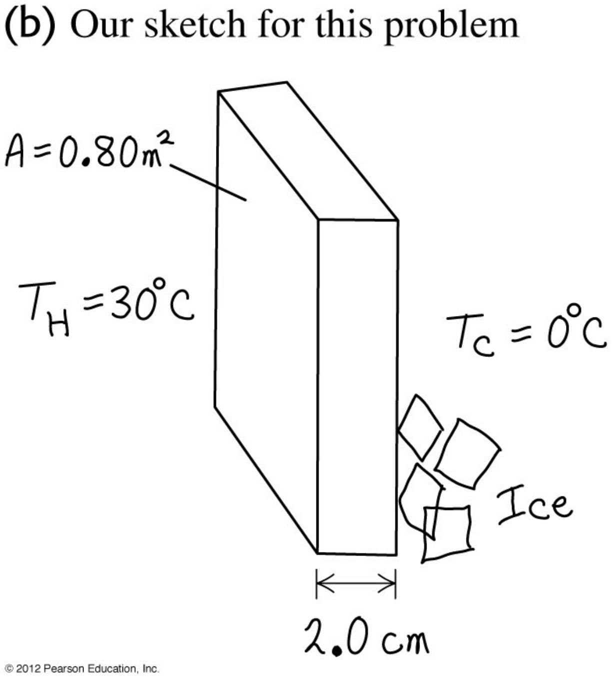


Figure 4: Calculate the heat flow through this wall of styrofoam assuming $k_{\text{sty}} = 0.027$ W/(m·K)

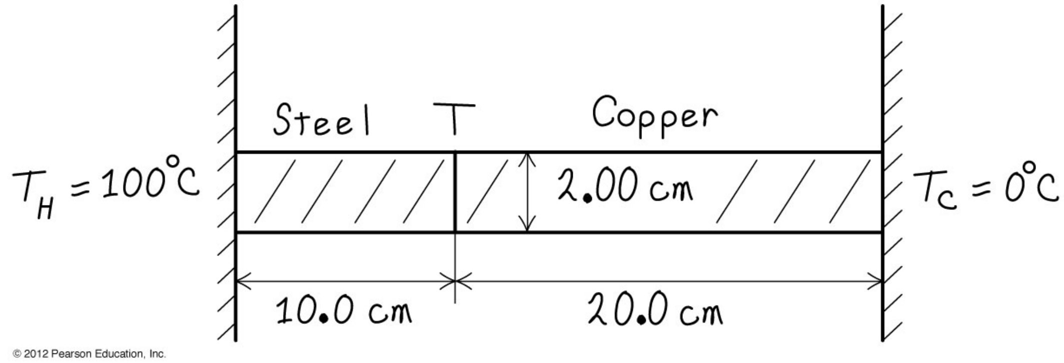


Figure 5: This is from Example 17.12 “Conduction through two bars.” Assume that both the steel (S) and the copper (Cu) are perfectly insulated around their side. (a) Calculate the temperature at their junction, and (B) Calculate the heat flow through the material. Notice how the R values ($R = L/k$) are additive when the materials are aligned end-to-end. $k_{\text{steel}} = 50.2\, W/m \cdot K$ and $k_{\text{copper}} = 385\, W/m \cdot K$

Convection

Heat transfer by convection occurs when a fluid, such as air or water, is in contact with an object whose temperature is higher than that of its surroundings. The warm fluid is less dense than the surrounding cooler fluid, so it rises because of buoyant forces.

Note: Convection cannot work without gravity. You cannot boil water in the International Space Station by using the principle of convection.

Radiation

Heat transfer by radiation occurs due to electromagnetic waves carrying energy from a "hot" source. For example, the radiant energy from the Sun is carried to the earth through the vacuum of the intervening space. There is an equation developed in modern physics which describes the rate of heat flow due to radiation, $H = \sigma A T^4$. The rate of heat flow due to radiation is proportional to T^4.

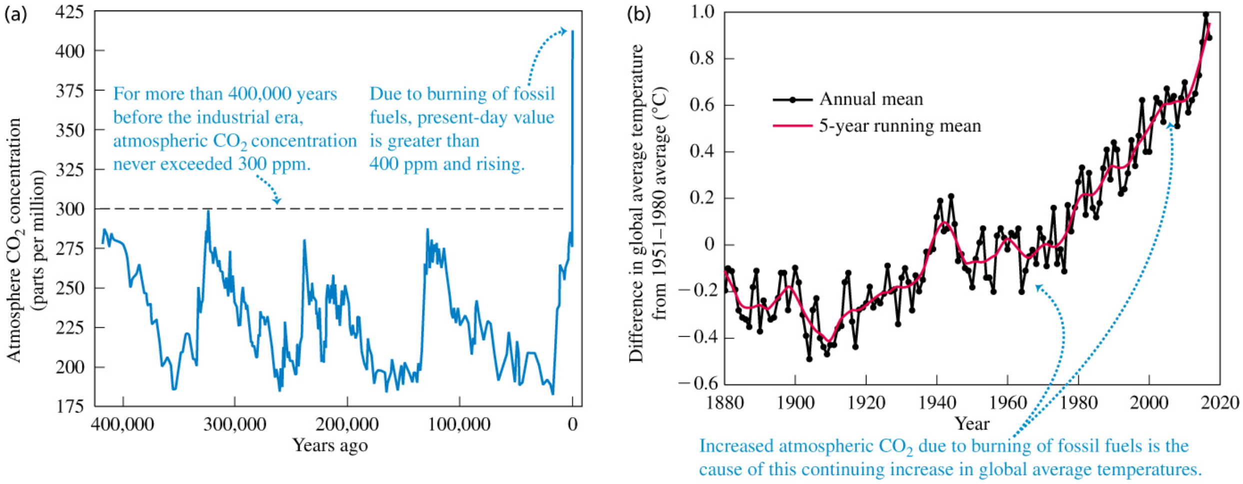
$$H = \frac{dQ}{dt} = e\,A\,\sigma\,T^4 \tag{5}$$

where σ is called the **Stefan-Boltzmann constant**:

$$\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2 \cdot K^4}$$

$$\sigma = \frac{2\pi^5\,k^4}{15c^2h^3} \qquad \text{You'll see this derived in Modern Physics.}$$

Ex. 106 The rate at which radiant energy from the sun reaches the earth's upper atmosphere is about 1.50 kW/m^2. The distance from the earth to the sun is 1.50×10^{11} m, and the radius of the sun is 6.96×10^{8} m. a) What is the rate of radiation of energy per unit area from the sun's surface? b) If the sun radiates as an ideal blackbody, what is the temperature of its surface?



For more than 400,000 years before the industrial era, atmospheric CO_2 concentration never exceeded 300 ppm.

Due to burning of fossil fuels, present-day value is greater than 400 ppm and rising.

Increased atmospheric CO_2 due to burning of fossil fuels is the cause of this continuing increase in global average temperatures.

Figure 6: (a) Due to humans burning fossil fuels, the concentration of carbon dioxide in the atmosphere is now more than 33% greater than in the pre-industrial era. (b) Due to the increased CO_2 concentration, during the past 50 years the global average temperature has increased at an average rate of approximate 0.18 C^o per decade.