Chapter 16
Sound and Hearing

1 Sound Waves

The most general definition of **sound** is that it is a longitudinal wave in a medium. The simplest sound waves are sinusoidal waves which have definite frequency, amplitude, and wavelength. The human ear is sensitive to waves in the audible range.

Frequency (Hz)	
0 - 20	infrasonic
20 - 20,000	audible
> 20,000	ultrasonic

The equation describing the displacement of molecules from their equilibrium position while a sound wave is propagating in the $+x$ direction is the familiar:

$$y(x,t) \; = \; A\cos(kx - \omega t)$$

N.B.: The y displacement of the molecules from their equilibrium position is along the x direction.

The change in volume ΔV is:

$$\Delta V = S\,(y_2 - y_1) \; = \; S\,[y(x + \Delta x, t) - y(x,t)]$$

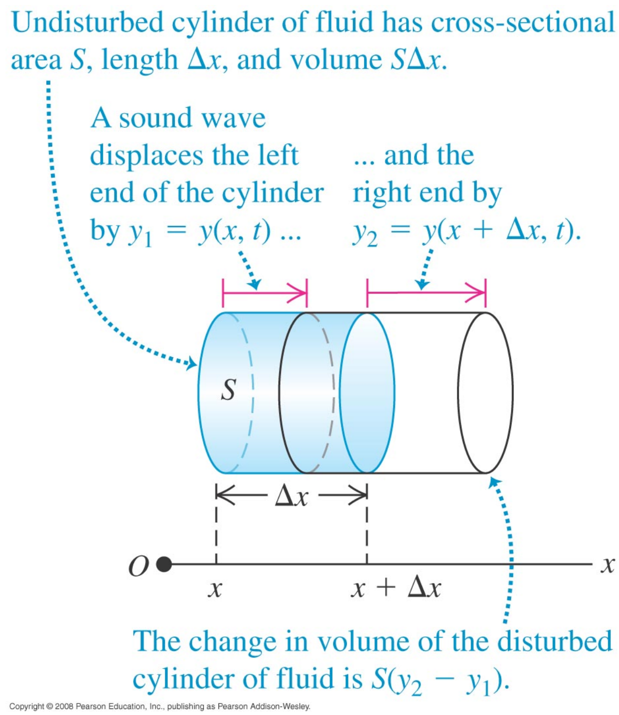
The fractional change in the volume of gas in the cylinder is:

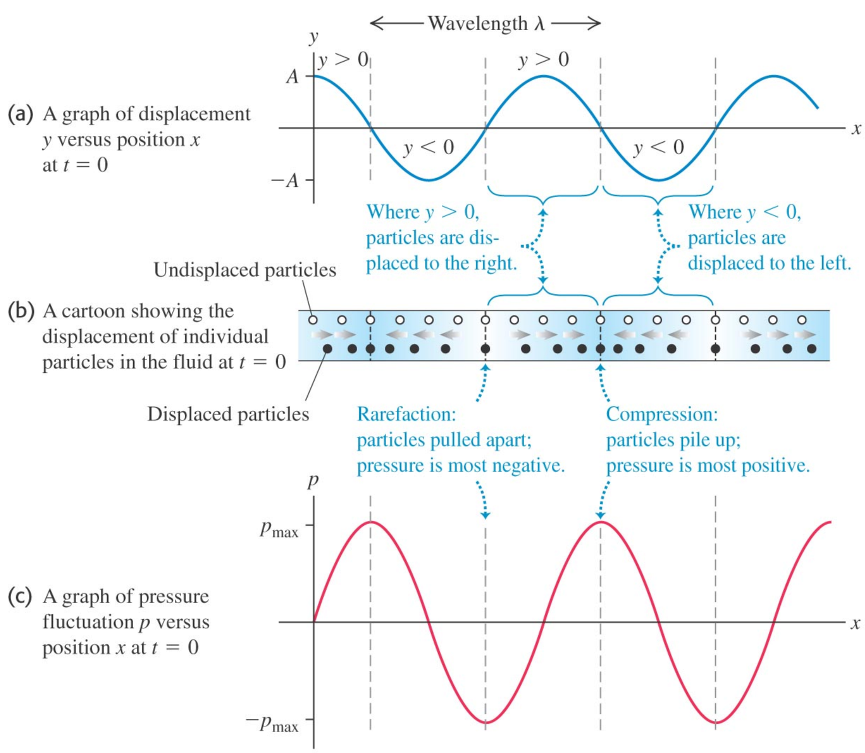
$$\frac{dV}{V} \; = \; \lim_{\Delta x \to 0} \frac{S\,[y(x+\Delta x, t) - y(x,t)]}{S\,\Delta x} \; = \; \frac{\partial y(x,t)}{\partial x}$$

However, using the bulk modulus $B = -p(x,t)/(dV/V)$, we can rewrite the previous equation as:

$$p(x,t) \; = \; -B\frac{\partial y(x,t)}{\partial x} \; = \; BkA\sin(kx - \omega t)$$

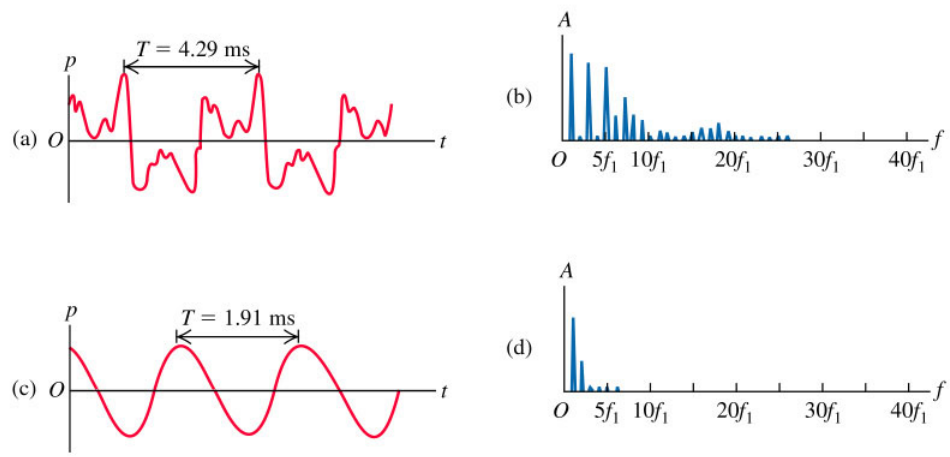
where $p_{\max} = BkA$.


Undisturbed cylinder of fluid has cross-sectional area S, length Δx, and volume SΔx.
A sound wave displaces the left end of the cylinder by y1 = y(x, t) ...
... and the right end by y2 = y(x + Δx, t).
S
Δx
O
x
x
x + Δx
The change in volume of the disturbed cylinder of fluid is S(y2 − y1).
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.


Wavelength λ
y
y > 0
y > 0
A
−A
y < 0
y < 0
x
(a) A graph of displacement y versus position x at t = 0
Where y > 0, particles are displaced to the right.
Where y < 0, particles are displaced to the left.
Undisplaced particles
(b) A cartoon showing the displacement of individual particles in the fluid at t = 0
Displaced particles
Rarefaction: particles pulled apart; pressure is most negative.
Compression: particles pile up; pressure is most positive.
p
pmax
−pmax
(c) A graph of pressure fluctuation p versus position x at t = 0
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

1.1 Perception of Sound Waves

The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. Perceptions of sound include loudness and pitch. In the figures below, you can see the waveforms, $p(t)$, produced by a clarinet (a) and by an alto recorder (c).



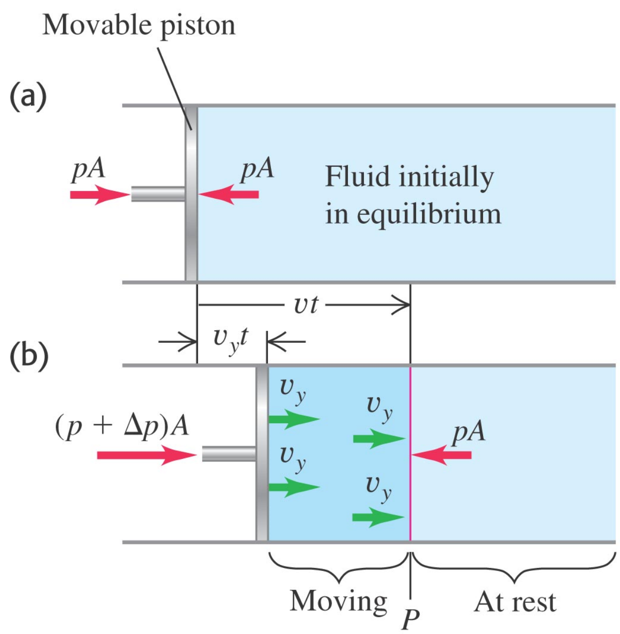
Ex. 1 Example 16.1 (Section 16.1) showed that for sound waves in air with frequency 1000 Hz, a displacement amplitude of 1.2×10^{-8} m produces a pressure amplitude of 3.0×10^{-2} Pa. a) What is the wavelength of these waves? b) For 1000-Hz waves in air, what displacement amplitude would be needed for the pressure amplitude to be at the pain threshold, which is 30 Pa? c) For what wavelength and frequency will waves with a displacement amplitude of 1.2×10^{-8} m produce a pressure amplitude of 1.5×10^{-3} Pa?

2 Speed of Sound Waves

A sound wave in a bulk fluid causes compressions and rarefactions of the fluid. A general expression for the speed of sound can be written as:

$$v = \sqrt{\frac{\text{(restoring force returning the system to equilibrium)}}{\text{(inertia resisting the return to equilibrium)}}}$$

In the previous chapter on wave motion, we calculated the power transmitted due to transverse wave motion. In this section and the next, we will look at how the energy (and power) are transmitted due to longitudinal motion of the medium that supports the wave motion.



$$\text{Longitudinal Momentum} = (\rho v t A) v_y \qquad (1)$$

where v is the speed of sound, and v_y is the speed of the piston pushing on the air molecules.

Also recall that the *bulk modulus* can be written as the stress/strain:

$$B = -\frac{\Delta p}{\Delta V/V_o} = \frac{-\Delta p}{-A v_y t/(A v t)}$$

$$\Delta p = B\frac{v_y}{v}$$

Likewise, let's calculate the *impulse* generated by the piston pushing on the gas (e.g., air molecules). Recall, that the impulse is defined to be $F\,\Delta t$.

$$\text{Longitudinal Impulse } = (\Delta pA)t = \frac{BAv_yt}{v} \tag{2}$$

Using the *momentum-impulse* theorem, we can compare Eqs. 1 and 2:

$$\rho vtAv_y = \frac{BAv_yt}{v} \qquad \text{or} \qquad v^2 = \frac{B}{\rho}$$

Thus, the speed of sound for a longitudinal wave in a fluid can be written as:

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{(speed of a longitudinal wave in a fluid)} \tag{3}$$

$$v = \sqrt{\frac{Y}{\rho}} \qquad \text{(speed of a longitudinal wave in a solid rod)} \tag{4}$$

Table 16.1 Speed of Sound in Various Bulk Materials

Material	Speed of Sound (m/s)
Gases	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
Liquids	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
Solids	
Aluminum	6420
Lead	1960
Steel	5941

2.1 Speed of Sound in Gases

The *bulk modulus* for a gas can be written as:

$$B = \gamma p_0$$

where p_0 is the equilibrium pressure of the gas and $\gamma = 1.4$ for diatomic molecules (e.g., oxygen and nitrogen molecules that make up most of our atmosphere).

The density ρ of a gas also depends on the pressure, which in turn depends on the temperature. However, the ratio B/ρ only depends on the absolute temperature. and can be written as:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where R = 8.314 J/mol·K, and T is the absolute temperature measured in *kelvin*.

Ex. 10 Show that the fractional change in the speed of sound (dv/v) due to a very small temperature change dT is given by $dv/v = \frac{1}{2}dT/T$. (Hint: Start with Eq. 16.10.) (b) The speed of sound in air at 20°C was found to be 344 m/s. What is the change in speed for a 1.0 C° change in air temperature?

3 Sound Intensity

In this section we will learn to write the sound intensity in terms of the displacement amplitude A or pressure amplitude p_{max}. In the previous chapter, where we calculated the power transmitted by transverse wave, we calculated the instantaneous power as the product of *force* times *velocity*. In this section, we calculate the *instantaneous intensity* as the product of (force/area)×*(velocity)*.

$$I(x,t) \;=\; p(x,t)\, v_y(x,t) \;=\; p_{max} \sin(kx - \omega t)\, A\omega \sin(kx - \omega t)$$

Combining these two terms we find the *instantaneous intensity* to be:

$$I(x,t) \;=\; BkA^2\omega\, \sin^2(kx - \omega t) \tag{5}$$

We would like to calculate the average intensity by taking the time average of $\sin^2(kx - \omega t)$, which, as we saw before, is $\frac{1}{2}$. So, the average intensity can be written as:

$$I_{\text{av}} = \frac{1}{2} B k A^2 \omega \qquad \text{(Average Intensity–for a sinusoidal wave)} \qquad (6)$$

This equation can be written in two other forms:

$$I_{\text{av}} = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 \qquad \text{and} \qquad I_{\text{av}} = \frac{p_{\text{max}}^2}{2\sqrt{\rho B}} \qquad \text{(for a sinusoidal wave)}$$

3.1 The Decibel Scale

The ear has a broad dynamical range of sensitivity to sound waves. A logarithmic intensity scale must be used to describe the human ear's response to sound waves, and this is called the *decibel* scale. The **sound intensity level** β is defined by the following equation:

$$\beta = (10\,\text{dB}) \log \frac{I}{I_o} \qquad \text{(definition of sound intensity level)} \qquad (7)$$

where $I_o = 10^{-12}$ W/m^2, the threshold of hearing at 1000 Hz.

Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

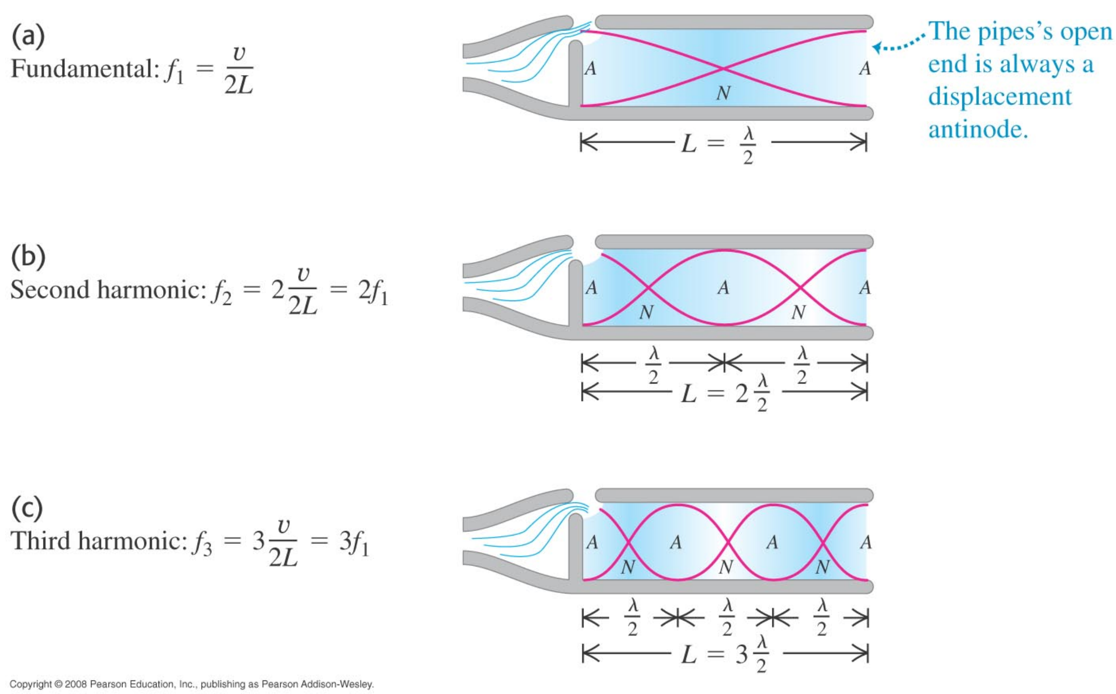
Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

Ex. 15 A sound wave in air at 20°C has a frequency of 320 Hz and a displacement amplitude of 5.00×10^{-3} mm. For this sound wave calculate the (a) pressure amplitude (in Pa); (b) average intensity (in W/m^2); (c) sound intensity level (in decibels).

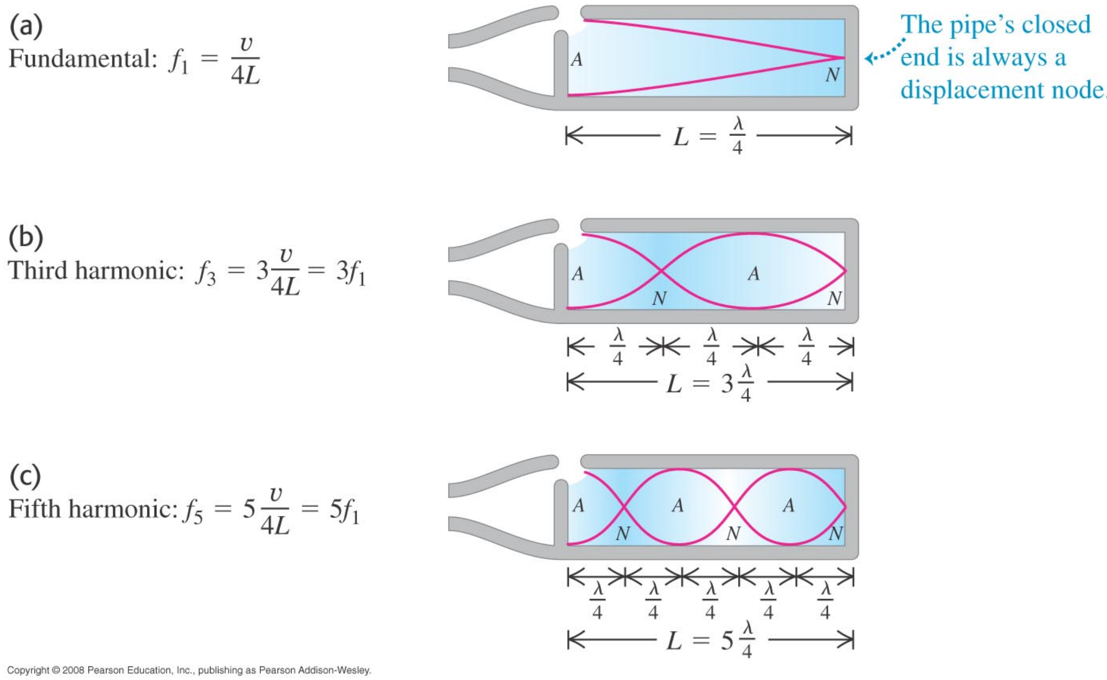
4 Standing Sound Waves and Normal Modes

It's possible to produce standing sound waves in a pipe of of length L. When sound waves reach the end of the pipe, they can be reflected from an *open* or *closed* ended pipe. Just as we saw transverse waves reflecting from a boundary in the previous chapter, longitudinal waves reflect from an *open* or *closed* ended tube in a similar manner.

Open Tube



Closed Tube



Let's consider what happens when traveling sound waves encounter the end of the tube. There are two possibilities:

1. the end of the tube is open
2. the end of the tube is closed

In the case where the tube is *open*, the displacement *antinode* is at the end of the tube.

$$\lambda_n = \frac{2L}{n} \qquad f_n = n\,\frac{v}{2L} \qquad (n = 1, 2, 3, \ldots) \qquad \text{“open”} \tag{8}$$

In the case where the tube is *stopped*, the displacement *node* is at the end of the tube.

$$\lambda_n = \frac{4L}{n} \qquad f_n = n\,\frac{v}{4L} \qquad (n = 1, 3, 5, \ldots) \qquad \text{“closed”} \tag{9}$$

Ex. 27 The longest pipe found in most medium-size pipe organs is 4.88 m (16 ft) long. What is the frequency of the note corresponding to the fundamental mode if the pipe is (a) open at both ends. (b) open at one end and close at the other?

5 Resonance

Ex. 30 You have a stopped pipe of adjustable length close to a taut 62.0-cm, 7.25-g wire under a tension of 4110 N. You want to adjust the length of the pipe so that, when it produces sound at its fundamental frequency, this sound causes the wire to vibrate in its second *overtone* with very large amplitude. How long should the pipe be?

6 Interference of Sound Waves

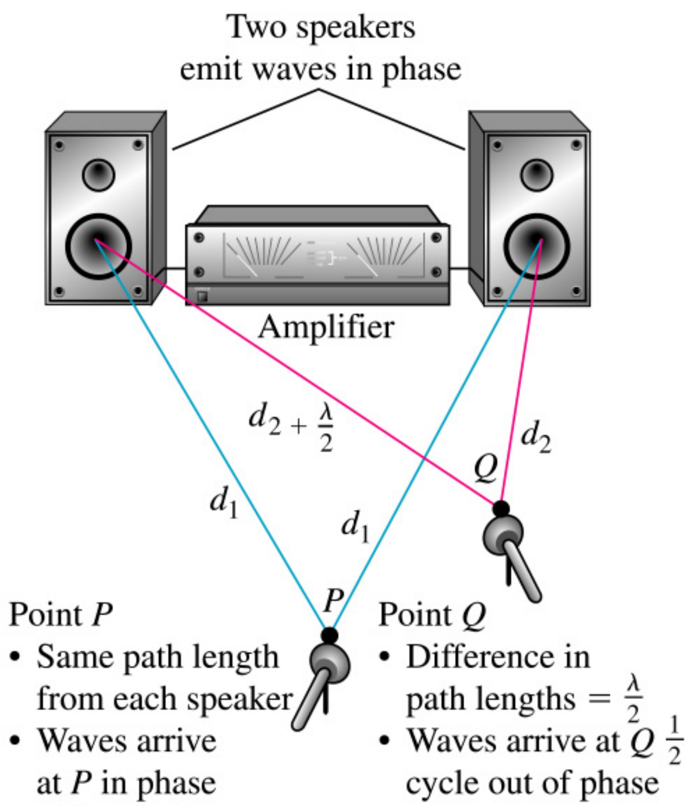
Imagine two loudspeakers driven from a common source and a listener hears the superposition of the two pressure waves coming from the speakers. Strictly speaking, the outgoing waves from the speaker are spherical waves whose pressure amplitudes do not remain constant, but instead decreases like $1/r$. The total pressure disturbance at point P is $\Delta p = \Delta p_1 + \Delta p_2$.

The type of interference that occurs at point P depends on the *phase difference* $\Delta\phi$ between the waves.

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda}$$

$$\Delta L = m\lambda \qquad (m = 0, 1, 2, \ldots) \qquad \text{constructive interference} \tag{10}$$

$$\Delta L = \left(m + \frac{1}{2}\right)\lambda \qquad (m = 0, 1, 2, \ldots) \qquad \text{destructive interference} \tag{11}$$



Such interference effects are most pronounced when the speakers are emitting one frequency only.

Ex. 33 Two loudspeakers, A and B (Fig. E16.33), are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.0 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q. What is the lowest frequency for which (a) *constructive* interference occurs at point Q; (b) *destructive* interference occurs at point Q?

6.1 Beats

Suppose we have two *pressure* waves with the same amplitude and slightly different frequencies $\omega_1 \approx \omega_2$.

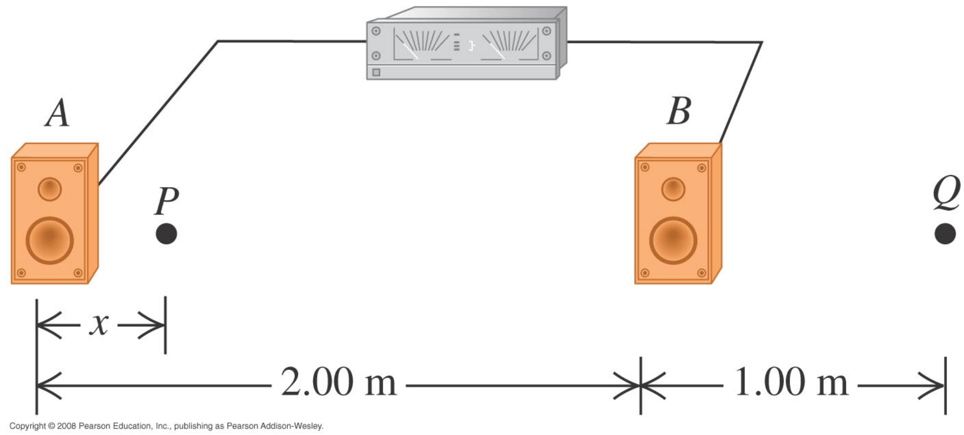


Figure 1: Figure 16.33 from University Physics 15th edition

Let's write the two pressure waves arriving at the same place as a function of time time in the following manner:

$$\Delta p_1(t) = \Delta p_m \sin(\omega_1 t) \qquad \text{and} \qquad \Delta p_2(t) = -\Delta p_m \sin(\omega_2 t)$$

Using the superposition principle, the resultant pressure is:

$$\Delta p(t) = \Delta p_1(t) + \Delta p_2(t) \; = \; \left[2\,\Delta p_m \sin\left(2\pi \frac{(f_1 - f_2)}{2} \right) t \right] \; \cos\left(2\pi \frac{(f_1 + f_2)}{2} \right) t \tag{12}$$

where we have used the familiar trigonometric identity:

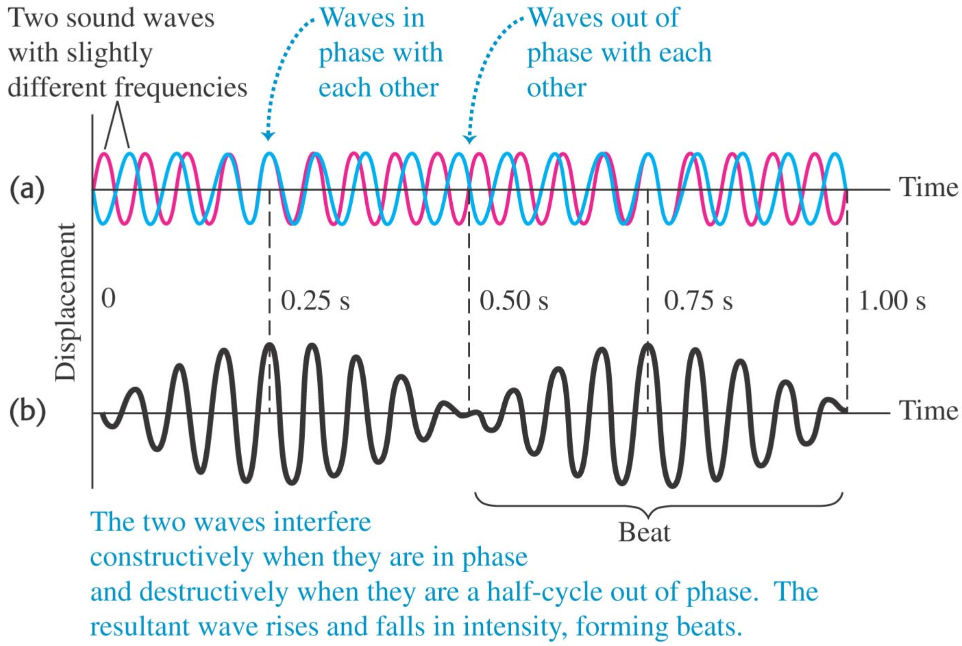
$$\sin A - \sin B \; = \; 2 \, \sin\left(\frac{A - B}{2} \right) \, \cos\left(\frac{A + B}{2} \right)$$

The *amplitude modulation* defined by the sine term goes from maximum to minimum (loud → loud) in π radians. This means that

$$\left(\frac{\omega_1 - \omega_2}{2} \right) t \; = \; \pi \qquad \text{or} \qquad t \; = \; \left(\frac{2\pi}{2\pi(f_1 - f_2)} \right) \; = \; \frac{1}{|f_1 - f_2|}$$

where t is the time between successive beats. Likewise, the frequency of beats (e.g., loud → loud), is

$$f_{\text{beat}} \; = \; \frac{1}{t} \; = \; |f_1 - f_2| \tag{13}$$



Ex 40: Two organ pipes, open at one end but closed at the other, are each 1.14 m long. One is now lengthened by 2.00 cm. Find the beat frequency they produce when playing together in their fundamentals.

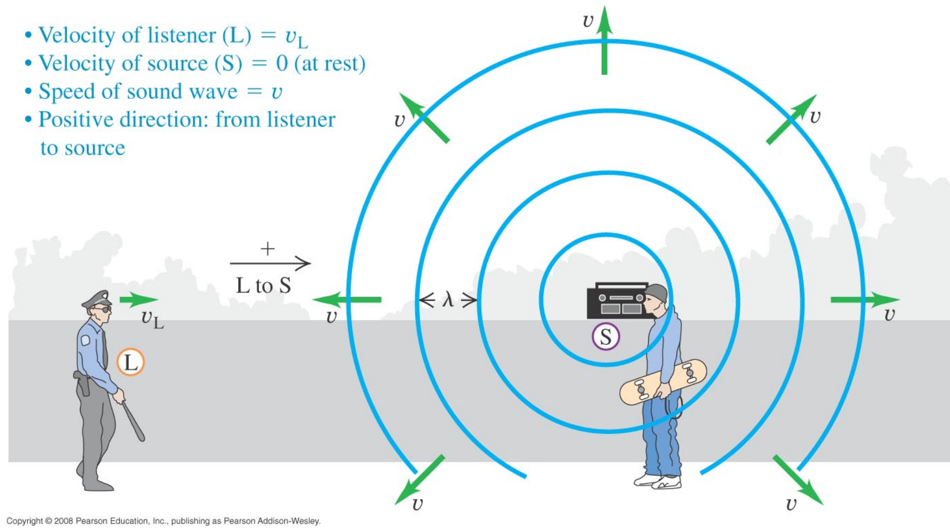
6.2 The Doppler Effect

The pitch heard by an listener changes when the sound source is moving with respect to the listener, and likewise, when the listener is moving with respect to the sound source.

Moving Listener, Source at Rest

The frequency heard (f') is the number of waves passing the listener *as if he were at rest* (vt/λ) plus the number of additional waves *as he moves towards the source* at velocity v_L ($v_o t/\lambda$) per unit time t.

$$f' = \frac{vt/\lambda + v_L t/\lambda}{t} = \frac{v + v_L}{\lambda} = \frac{v + v_L}{v/f} = f\left(\frac{v + v_L}{v}\right)$$



In general, we can write the frequency heard by a moving listener with the source at rest as:

$$f' = f\left(\frac{v \pm v_L}{v}\right) \qquad \text{(+ sign, if the listener is moving toward the source)} \quad (14)$$

Moving Source, Listener at Rest

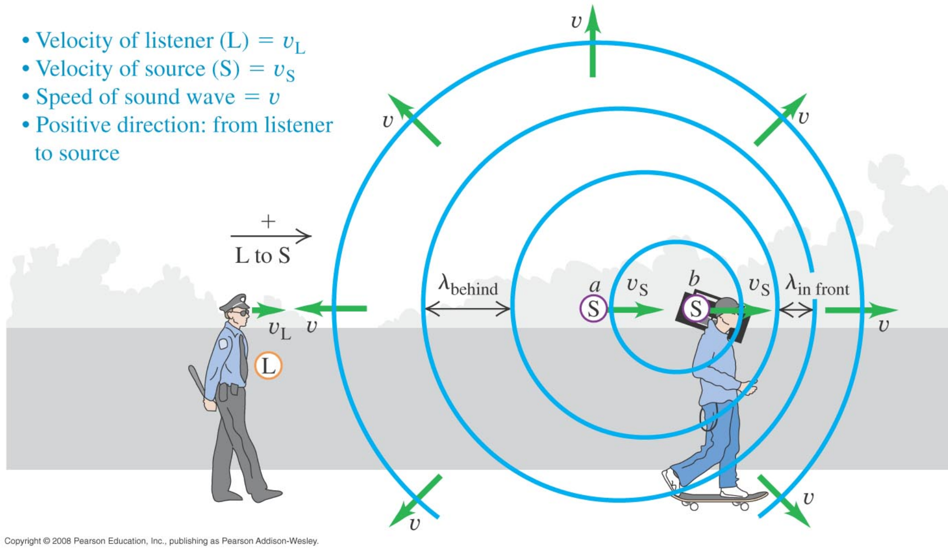
The pitch of the sound heard by the listener at rest (f') is the speed of sound divided by the *compressed* wavelength (λ') if the source is moving *toward* the stationary listener. As the source approaches the listener, we have

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - v_s)T} = f\left(\frac{v}{v - v_s}\right)$$

In general, we can write the frequency heard by the stationary listener as the source moves *towards/away* the listener as:

$$f' = f\left(\frac{v}{v \mp v_s}\right) \qquad \text{(− sign, if the source is moving toward the listener)} \quad (15)$$

If both the source and the listener are moving



If both the source and the listener are moving, Eqs. 14 and 15 can be combined into one equation.

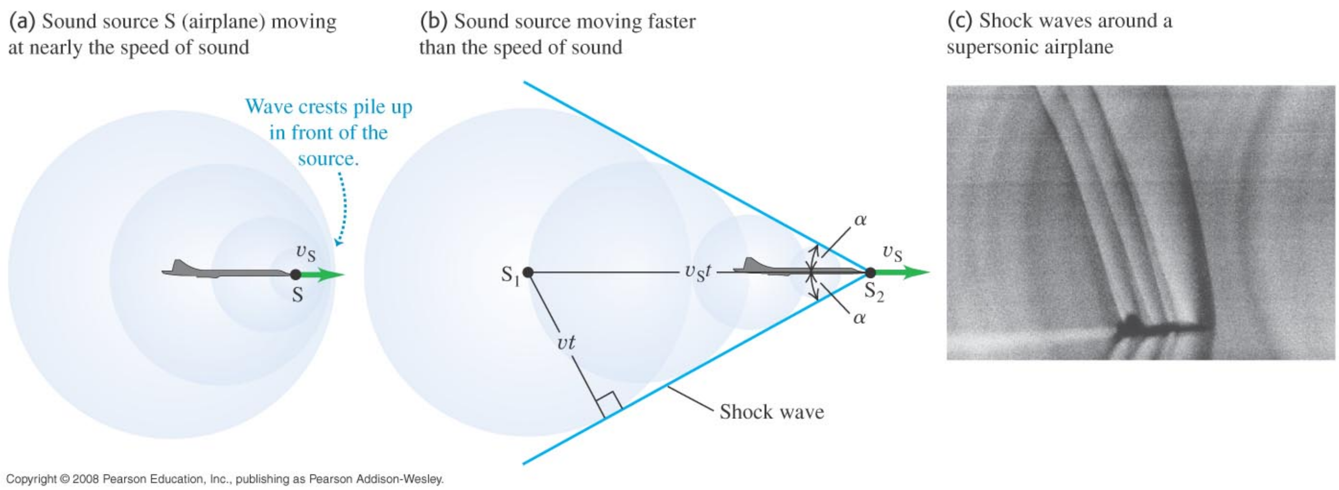
$$f' = f \left(\frac{v \pm v_L}{v \mp v_s} \right) \tag{16}$$

N.B. v_L and v_s are measured with respect to the medium carrying the sound waves, in this case the air.

Ex. 44 Moving Source vs. Moving listener. a) A sound source producing 1.00 kHz waves moves toward a stationary listener at $\frac{1}{2}$ the speed of sound. What frequency will the listener hear? b) Suppose instead that the source is stationary and the listener moves toward the source at $\frac{1}{2}$ the speed of sound. What frequency does the listener hear? How does your answer compare to that in part (a)? Explain on physical grounds why the two answers differ.

7 Shock Waves

$$\sin\alpha = \frac{vt}{v_s t} = \frac{v}{v_s} \qquad \text{(shock wave)} \tag{17}$$



Ex. 53 A jet plane flies overhead at Mach 1.70 and at a constant altitude of 1250 m. a) What is the angle α of the shock-wave cone? b) How much time after the plane passes directly overhead do you hear the sonic boom? Neglect the variation of the speed of sound with altitude.

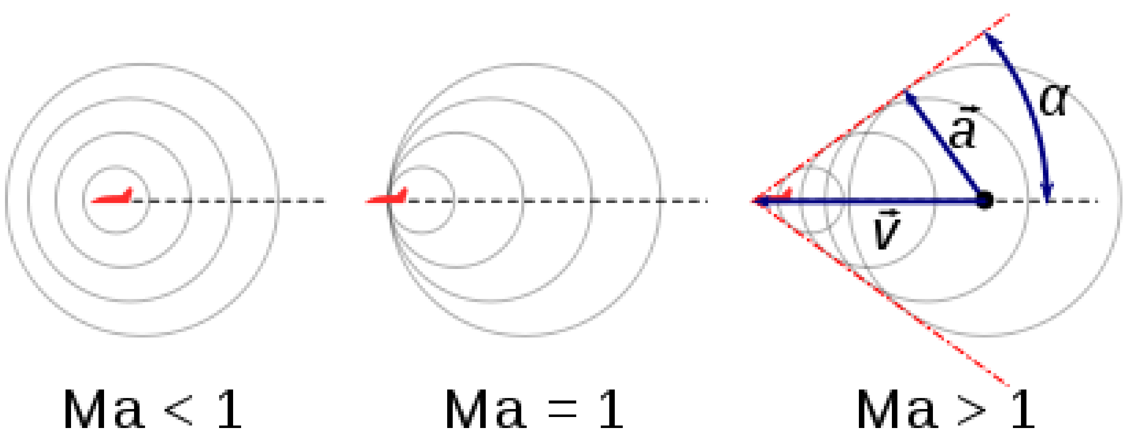


Figure 2: Illustration of the shock wave created when the Mach number is <1, =1, or >1.

Ex. 63 The sound source of a ship's sonar system operates at a frequency of 18.0 kHz. The speed of sound in water (assumed to be at a uniform 20°C) is 1482 m/s. (a) What is the wavelength of the waves emitted by the source? (b) What is the difference in frequency between the directly radiated waves and the waves reflected from a whale traveling directly toward the ship at 4.95 m/s? The ship is at rest in the water.