THE ZEEMAN EFFECT

OBJECT: To measure how an applied magnetic field affects the optical emission spectra of mercury vapor and neon -- the Zeeman Effect.

APPARATUS: Spex 1000M 1 meter grating spectrometer system including: 1800 lines/mm holographic grating, "Quickscan" thermoelectrically cooled linear diode detector array, Hamamatsu R928P side-window photomultiplier photon detector w/ high voltage supply and preamplifier-amplifier, Jabon Yvon-Spex "Spectramax" control and data acquisition software and Dell 486 computer; Hg and Ne discharge tubes; Gas discharge power supply; Penray miniature Hg discharge tube and power supply; Iron core electromagnet, Varian 6121 30-ampere Variac magnet power supply; RFL Industries model 904 Gaussmeter with Hall probe; Fiber optic feed/collimator with rotatable polarizer.

Safety: Beware of the high voltage on gas discharge tube power supply. It can give you a painful shock. Also, to avoid overheating the magnet coils, do not run the electromagnet above 5 amperes for more than 20 minutes. [Occasionally feel the coils. If they feel very warm, turn of the magnet and let them cool for 30 minutes.]

THEORY: The Zeeman effect is the name given to the splitting of the energy levels of an atom when it is placed in an externally applied magnetic field.

The splitting occurs because of the interaction of the magnetic moment μ of the atom with the magnetic field **B** slightly shifts the energy of the atomic levels by an amount

$$\Delta E = - \mu.\textbf{B}. \tag{1}$$

This energy shift depends on the relative orientation of the magnetic moment and the magnetic field.

Nuclear magnetic resonance (NMR) and electron spin resonance (ESR) both depend on the Zeeman splitting of a single energy level within the atom. In the case of NMR, the splitting depends on the orientation of the nuclear moment relative to a large static applied magnetic field, and NMR is induced by applying a small alternating magnetic field of exactly the energy given by Eq. (1) (typically with a frequency in the 20 MHz range for a conveniently sized static magnetic field) to induce the nuclear moment to flip its orientation in the static field. ESR depends on the orientation of the electron magnetic moment and requires a much more energetic alternating field (typically 10 GHz) to induce resonance.

In this experiment you will study the optical Zeeman effect, which is a little more complicated than either NMR or ESR because it involves two energy

levels within the atom. In the optical Zeeman effect, atoms are excited to a level above the ground state by, for example, collisions with electrons in an electrical discharge (such as the commonly used fluorescent light bulb). When they return to the ground state, they emit the extra energy as a visible photon whose energy corresponds to the difference in energy between the excited and ground states. If a magnetic field is applied, both the ground state and the excited state energies experience Zeeman splitting. So photons of slightly different energies will be emitted when various excited atoms return to the ground state. We discuss below how to calculate the number of lines that a single spectral line is split into by a magnetic field and how that splitting depends on the magnitude of **B**.

Zeeman discovered the effect in 1896 and subsequently explained it using the Bohr picture of the atom. In the Zeeman-Lorentz explanation, the electron moving in a magnetic field experiences a Lorentz force that slightly changes the orbit of the electron and hence its energy. The change in energy depends on the orientation of the orbit. If the plane of the orbit is perpendicular to the magnetic field ΔE is either positive or negative depending upon whether the motion of the electron is clockwise or counter-clockwise. If the field lies in the plane of the orbit, the net Lorentz force is zero (averaged over one orbit) and $\Delta E = 0$. This reasoning thus predicts that the spectral emission line will split into three lines when a field is applied. More detailed arguments predict the same result for an arbitrary orientation of the plane of the orbit. A more modern argument, not using forces, can be made by noting that the orbital motion of the electron (angular momentum L) produces a magnetic moment μ_ℓ. Equation 1 can then be used to calculate how the Zeeman splitting will depend on the orientation of the angular momentum relative to the magnetic field.

In practice this "normal" three-line Zeeman effect is not commonly observed; If a spectrometer with high resolution is used, it is frequently found that the magnetic field splits the spectral lines into more than three components and even when the three line pattern is observed, the splitting increases more rapidly with applied field than predicted by the Zeeman-Lorentz theory. This "anomalous" Zeeman effect was not explained for a number of years until it was realized that, in addition to orbital angular momentum, electrons possess spin angular momentum S and an associated magnetic moment μ_s. The reason orbital and spin angular momenta have different Zeeman splittings is that electron spin has twice the magnetic moment than that due to orbital motion with the same angular momentum. In particular, an electron (charge e, mass m) traveling in a circular orbit in a plane perpendicular to the z axis has a magnetic moment μ_{ℓ_z} that is related to the orbital angular momentum L by the relation (see Halliday, Resnick and Walker[1] for a derivation)

$$\mu_{\ell z} = \frac{e}{2m} L_z. \tag{2}$$

On the other hand, for electron spin S electron spin it is experimentally observed (and theoretically predicted using quantum electrodynamics) that

$$\mu_{sz} = g \frac{e}{2m} S_z \tag{3}$$

where g = 2.0023. For the purposes of this discussion we take the g-factor to be exactly 2. For an electron that has both spin angular momentum S, orbital angular momentum L, and total angular momentum J, we can write

$$\mu_{Jz} = g_L \frac{e}{2m} J_z \tag{4}$$

where J_z is the z-component of the total angular momentum. g_L is called the Lande' g-factor and is given by

$$g_L = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}. \tag{5}$$

The derivation of Eqs. (4) and (5) is complicated but is generally covered in introductory quantum mechanics. Lande' first derived it semi-empirically. Notice that when S = 0, J = L and $g_L = 1$ so that Eq. (4) reduces to Eq. (2). On the other hand, when L = 0, J = S and $g_L = 2$ so that Eq. (4) reduces to Eq. (3). For other values of S and L, Eq. (5) gives intermediate values for g_L. For example, if L = S then $g_L = 3/2$.

We can now combine Eqs. (1) and (4) to obtain the Zeeman splitting. If the magnetic field is in the z direction,

$$\Delta E = -g_L \frac{e}{2m} J_z B. \tag{6}$$

Quantum mechanics tells us that the total angular momentum is quantized according to $J_z = m_j \hbar$ where $m_j = 0, 1, 2, \dots$. Thus

$$\Delta E = -g_L \frac{eh}{4\pi m} m_j B = -g_L \mu_B m_j B \tag{7}$$

where $\mu_B = .9274 \times 10^{-23} J/T = 5.788 \times 10^{-5} eV/T$ is called the Bohr magneton. The light that is emitted from the gas in a discharge tube is generated when electrons make transitions from exited states of the atom to the ground state. When an electron makes such a transition, it emits a single photon which

carries away an angular momentum of $\hbar$, which means transitions can only occur between states where J_z differs by only 0 or ± 1 $\hbar$ (or $\Delta m = m_j - m_j' = 0, \pm 1$, where the prime refers to the initial state). If the transition is between levels with the same m_j then the photon's energy is unshifted by the magnetic field. But if the change in m_j is ± 1, then the change in photon energy is

$$\Delta h\nu = (g_L m_j - g_L' m_j')\mu_B B = g_{eff}\mu_B B \qquad (8)$$

where $\Delta m = m_j - m_j' = \pm 1$. g_{eff} is the effective g-factor for the transition. The resulting spectrum is shown in Fig. 1 for the case $g_L = g_L' = 1$, the "normal" Zeeman effect. (g_L' is the Lande' g-factor for the upper level and g_L for the lower.) Note that transitions from the other Zeeman split excited states will give exactly the same three line pattern since the magnitude of the splitting of the excited and ground states is the same. In fact, you will get a three line pattern as long as $g_L = g_L'$. But g_{eff} will be different from the "normal" Zeeman effect. If $g_L \neq g_L'$ you will observe more than three lines in the spectrum. To
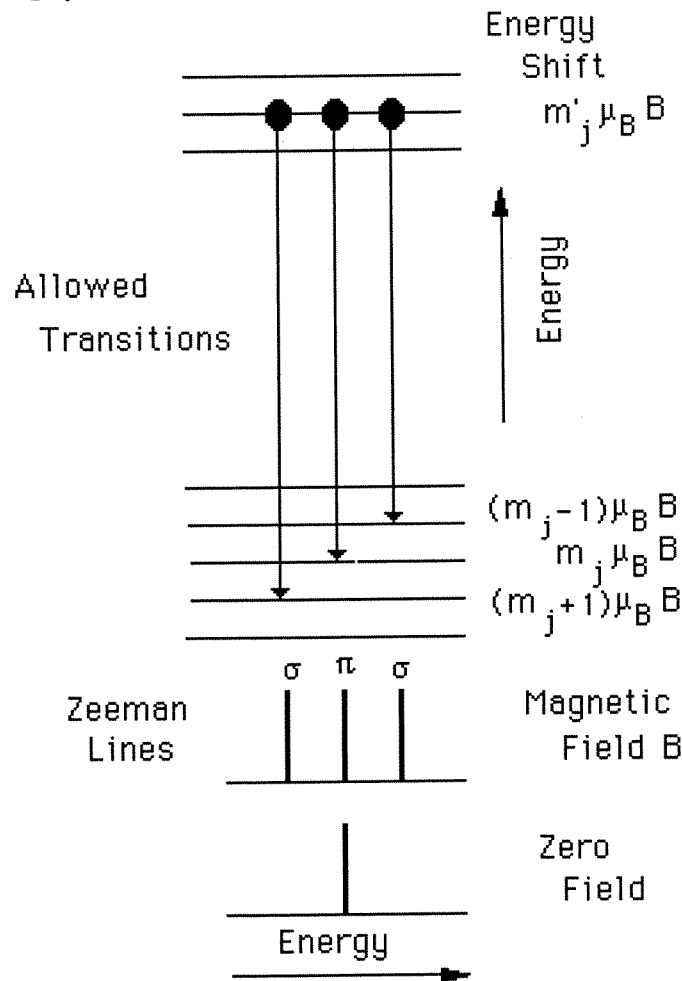

Figure 1 Optical spectrum for $g_L = g_L' = 1$, the "normal" Zeeman Effect

help understand this "anomalous" pattern we will study in the next section the spectrum of mercury. Before we do this, we need to discuss the polarization of the emitted light. In the spectrographic jargon, **photons emitted in a transition where** $\Delta m = \pm 1$ **are labeled** σ **lines** and are circularly polarized when observed parallel to the magnetic field and **linearly polarized perpendicular to the field when viewed at right angles to the field**. While **photons emitted when** $\Delta m = 0$ **are labeled** π **lines and plane polarized with the direction of polarization parallel to the field**. Thus the different components of the Zeeman spectrum shown in Fig. 1 will have the indicated polarizations. To observe light emitted parallel to the field would require a magnet with a hole through the pole pieces, which we do not have. When the light is observed perpendicular to the field the π and σ radiation will have the polarizations shown in the sketch in Fig. 2. Fowles[4] gives an excellent theoretical explantion for this polarization behavior (see especially Figs. 8-10 and 8-11).
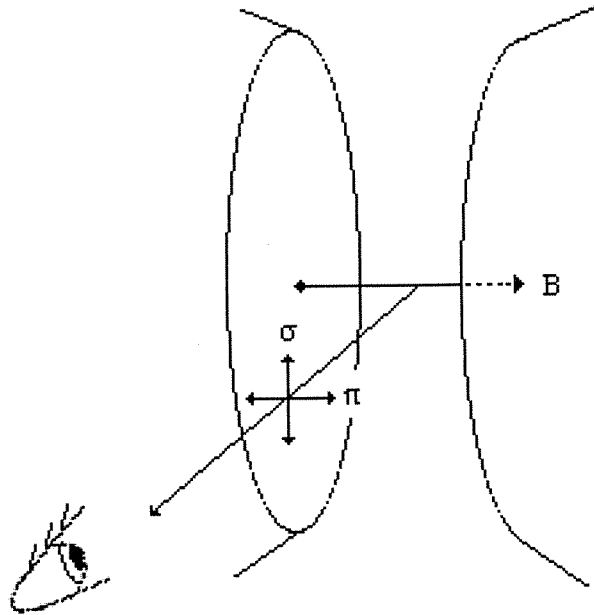

Figure 2. Polarization of π and σ radiation for the Zeeman Effect.

The Zeeman Effect in Mercury: For the ground state of mercury the two valence electrons are in the $(6s)^2$ configuration (two electrons in n=6, ℓ=0 single particle states). Because of their mutual electrical repulsion the two electrons do not move independently and three quantities -- the total spin angular momentum **S**, the total orbital angular momentum **L**, and the total angular momentum **J** -- are constants of the motion designated by quantum numbers S, L, and J respectively. The electron states are labeled with the spectroscopic notation $^{2S+1}L_J$. The value of L is denoted by S for L = 0, P for L = 1, etc. The ground state in mercury is then $^1S_0$ (This is a singlet state, S=0,

because the 6s electrons are in identical orbital states, the spins must be antiparallel according to the Pauli Exclusion Principle.)

As shown in Fig. 2, the next levels above the ground state in mercury are a triplet of levels -- $^3P_0$, $^3P_1$, and $^3P_2$ -- corresponding to single electron states (6s6p) where the electron spins are parallel. There is a higher singlet $^1P_1$ state with spins antiparallel and still higher a 6s7s state. In the Franck-Hertz experiment you study transitions between the ground state and the $^3P_1$, which involves ultraviolet energies. The visible light you see coming from a discharge tube rises from an electron excited to the 7s state dropping down to a 6p state. Figure 3 shows the three most intense visible spectral lines.
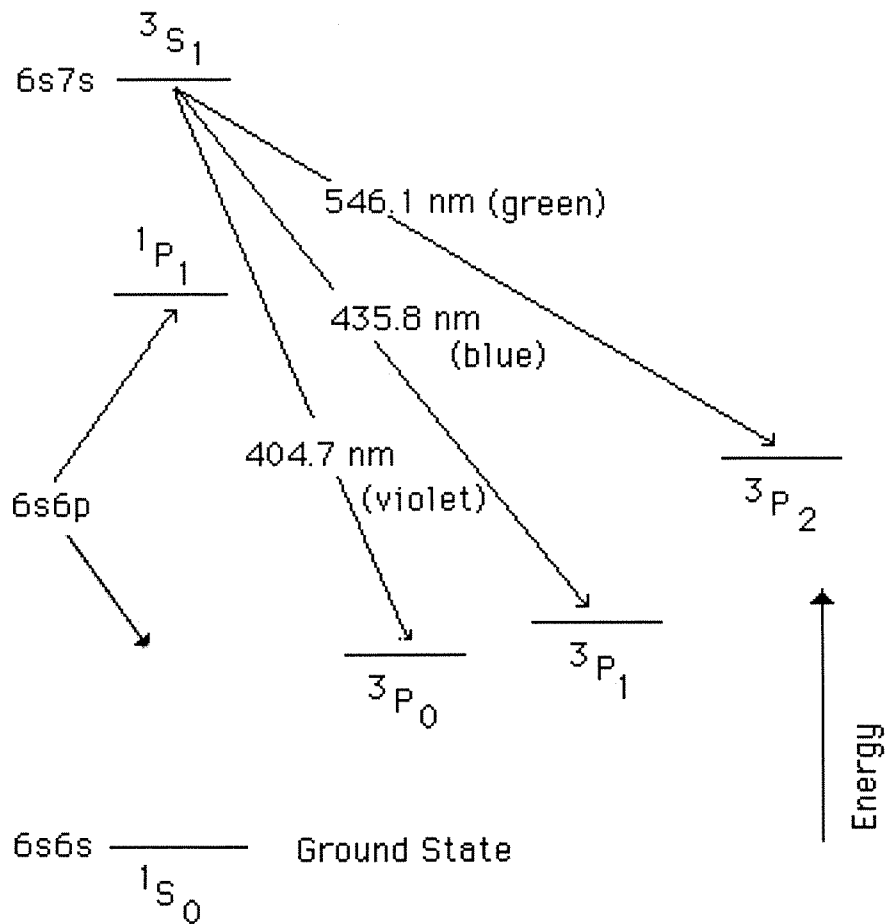

Figure 3: Energy Levels of Atomic Mercury
(not to scale)

To calculate the Zeeman spectrum of, for example, the blue line (435.8 nm) we first use Eq. (5) to calculate the Lande' g-factor for the $^3S_1$ and $^3P_1$ states:

$^3S_1$: J=1, L=0, and S=1 $\Rightarrow g'_L = 2$ (as expected for a pure spin state)
$^3P_1$: J=1, L=1, and S=1 $\Rightarrow g_L = 3/2$.

Then Eq. (8) tells us that we will see that the spectral line at $h\nu_0$ corresponding to the wavelength $\lambda = 435.8$ nm will be split into several lines by:

$$\Delta h\nu = g_{eff}\mu_B B = (\frac{3m_j}{2} - 2m_j')\mu_B B,$$

where $m_j = 0,\pm 1$, $m_j' = 0,\pm 1$, and $\Delta m = m_j - m_j' = 0,\pm 1$. Table 1 lists all possibilities. We see that the 435.8 nm line is split into six lines as shown in Fig 4. However, as discussed in the next section, if the spectrum is measured without adequate spectrometer resolution the pattern may appear as a three-line spectrum whose lines broaden in width as well as shift position as the field increases.

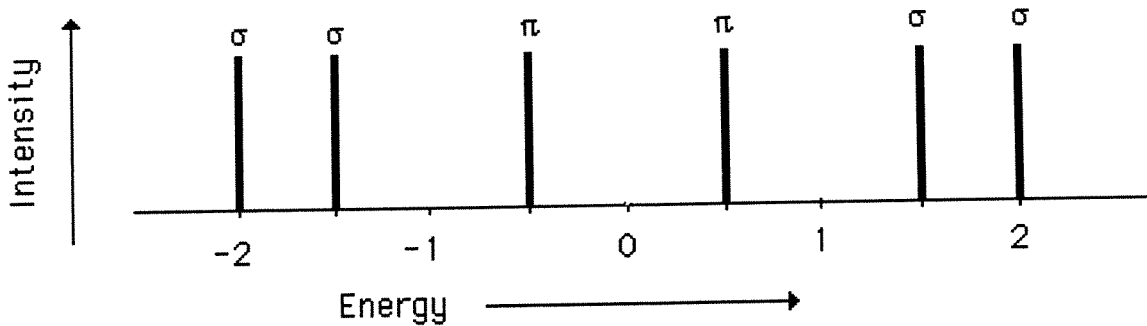| Table 1: Allowed Transitions 7s6s ^{3}S$_1$ to 6p6s ^{3}P$_1$				
m_j'	m_j	Δm	polarization	$g_{eff} = \Delta h\nu / \mu_B B$
1	1	0	π	-1/2
0	1	1	σ	3/2
-1	1	2		
1	0	-1	σ	-2
0	0	0		
-1	0	1	σ	2
1	-1	-2		
0	-1	-1	σ	-3/2
-1	-1	0	π	1/2

Figure 4. Zeeman splitting of the 7s6s ^{3}S$_1$ to 6p6s ^{3}P$_1$ transition in mercury (in energy units of $\mu_B B$).

Similar calculations for the 7s6s ^{3}S$_1$ --> 6p6s ^{3}P$_2$ transition show a nine line pattern, while the 7s6s ^{3}S$_1$ --> 6p6s ^{3}P$_0$ transition shows a triplet that might be interpreted as the "normal" Zeeman effect except the splitting varies as $\pm 2\mu_B B$, while one would expect it to vary as $\pm \mu_B B$ for the "normal" effect ($g_L = g_L' = 1$).

* but not m_j' = 0 to m_j = 0 if ΔJ = 0

A similar "normal" Zeeman triplet is obtained for the strong yellow transition at 585.3 nm in neon. This case is more difficult to calculate since the excitation occurs when an electron moves from a filled p-shell to a higher excited state, leaving behind a hole -- $1s^2 2s^2 2p^6 \rightarrow 1s^2 2s^2 2p^5 nx^1$. L-S coupling does not occur and the Lande' g-factor, Eq. (5), does not apply.

Effect of instrument resolution: There are several factors that determine the intrinsic width of the spectral lines such as the lifetime of the excited state and the effect of collisions within the discharge tube. But with the apparatus available in our laboratory the observed linewidth is determined by the resolution of the spectrometer. This resolution is determined by factors such as the number of grooves per mm in the diffraction grating and the width of the entrance and exit slits. The spectrometer you will be using has a resolution of 0.06 Å = 0.006 nm, where the resolution is defined to be the full width of the spectral line at the point where its intensity drops to one-half of its peak value. To understand the effect of instrumental resolution, we calculate the wavelength splitting for a Zeeman energy of $\Delta h\nu = g_{eff}\mu_B B$. Using $\lambda\nu = c$, where c is the speed of light, we find

$$|\Delta\lambda| = \left(\frac{c}{\lambda^2}\right)^{-1}\Delta\nu. \qquad (9)$$

Thus

$$|\Delta\lambda| = g_{eff}\frac{\lambda^2}{hc}\mu_B B = 4.668\times10^{-8}g_{eff}\lambda^2 B \qquad (10)$$

where wavelengths are in nm and B is in Tesla. The maximum value of B that you will be able to produce in the lab is about 1 T. So for the 435.8 nm 7s6s ^{3}S$_1$ to 6p6s ^{3}P$_1$ transition we find $|\Delta\lambda| = 0.0089 g_{eff}$ nm which is only moderately larger than the resolution. Figure 5 shows a simulation of the spectrum of the due to the σ components of the 7s6s ^{3}S$_1$ to 6p6s ^{3}P$_1$ transition. In the figure the intensity of the σ four components given in Table 1 (g_{eff} = -2, -3/2, +3/2, and 2) are added assuming a Lorentzian line shape for the individual lines:

$$I(\lambda) = \frac{1}{\pi\delta\lambda}\frac{1}{1 + \frac{(\lambda - \lambda_o)^2}{(\delta\lambda)^2}} \qquad (11)$$

where λ_o is the wavelength of the unsplit line and $\delta\lambda$ is the width caused by instrumental resolution (the resolution equals $2\delta\lambda$). [The factor $\pi\delta\lambda$ normalizes the intensity so that $\int_{-\infty}^{\infty} I(\lambda)d\lambda = 1$.] The simulation shows that the resolution will not be good enough to resolve the predicted four line σ pattern for the available magnetic field. Note that the peak intensity drops by a factor of more than three and the line width increases, which will affect the

signal-to-noise. Also note that the unresolved peak of the two lines falls between their splitting for $g_{eff} = 2$ and $g_{eff} = 3/2$. Thus the measured effective g-factor should fall at about 1.75 and you will be able to clearly recognize that this is an example of anomalous Zeeman effect even though you will not be able to resolve the predicted spectrum. Likewise the π lines will not be resolved, but you should observe the line to broaden as the magnetic field is increased. In contrast the pattern for the 7s6s $^3S_1$ to 6p6s $^3P_0$ transition, which involves only two σ lines will be fully resolved and should agree with the theoretical prediction, $g_{eff} = 2$.
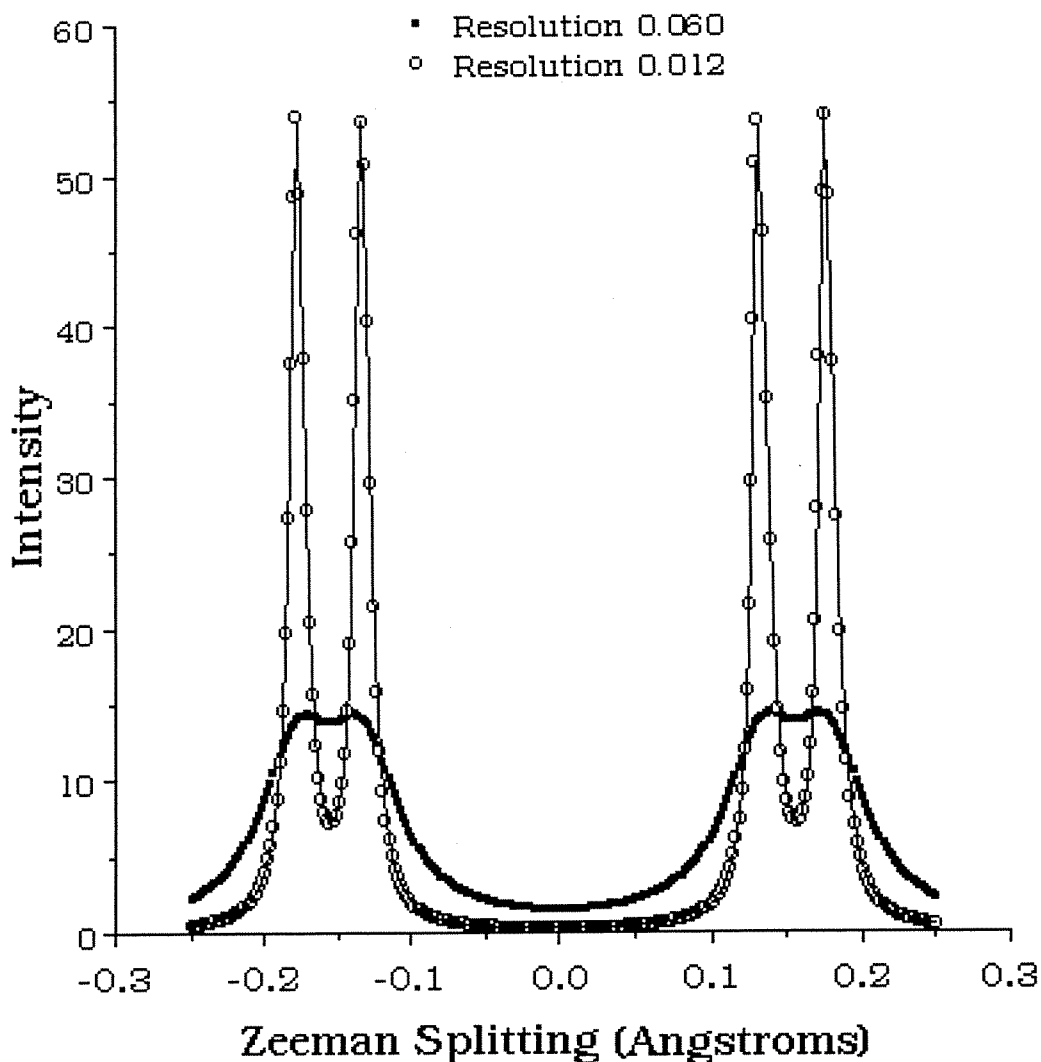

Figure 5. Dependence upon spectrometer resolution of the predicted Zeeman spectrum of the 7s6s $^3S_1$ to 6p6s $^3P_1$ transition in mercury for an applied magnetic field of 1 Tesla (σ polarization)

APPARATUS: The spectrograph that you will be using is fully computer controlled and easy to operate. Although you can usually get good results without understanding how it works, it is important to understand the principle of operation:

The **Czerny-Turner scanning spectrometer**, which you will be using in this experiment, uses a holographic 1800 groves/cm diffraction grating to disperse incoming light by wavelength into a spectrum. The direction of incidence of the light and the direction of observation, Fig. 6, are both fixed. Incident light

curved refocusing mirror   curved collimating mirror

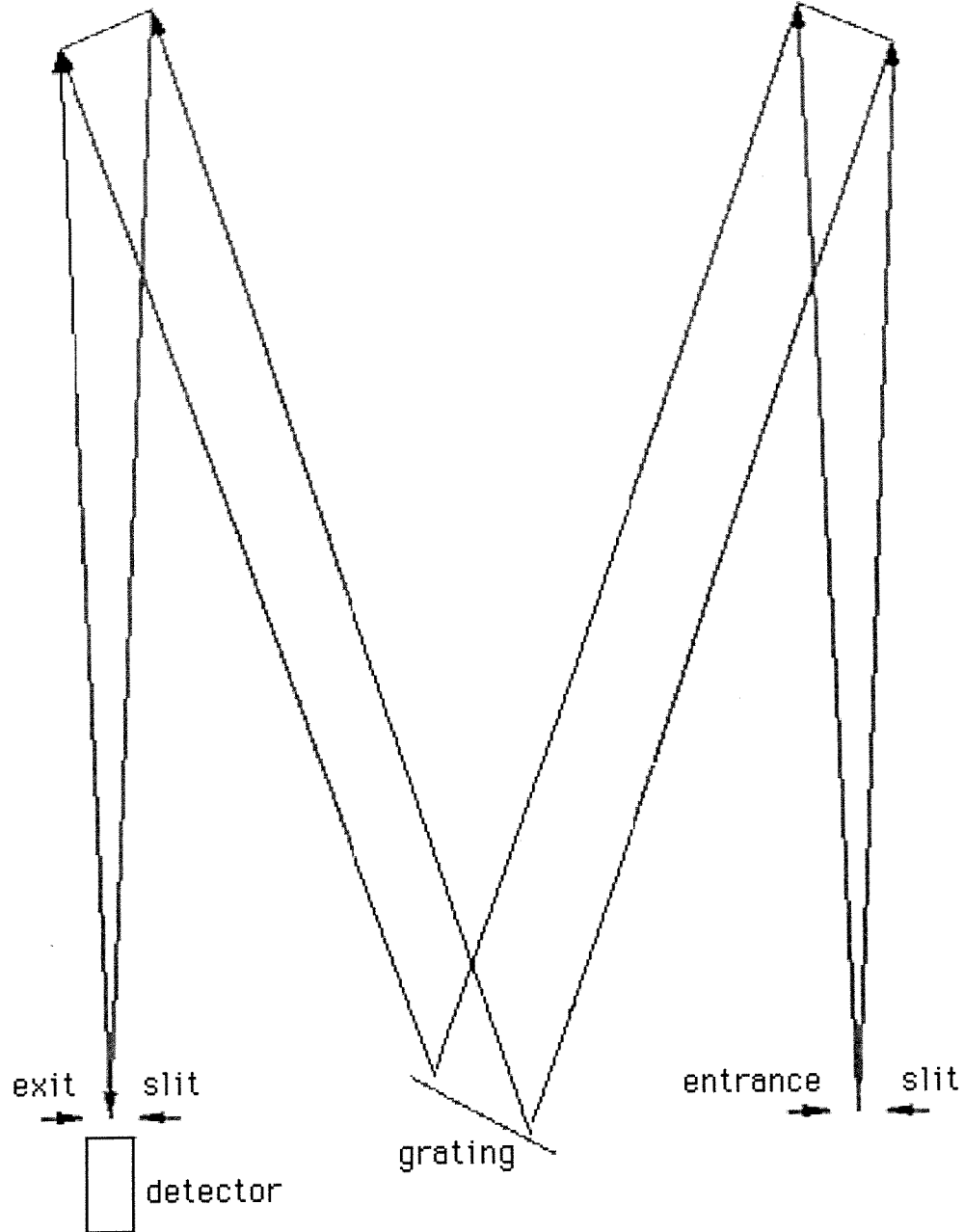exit   slit   entrance   slit

grating

detector

Figure 6. Czerny-Turner Scanning Spectrometer

containing a mixture of wavelengths passes through the entrance slit and strikes the first (collimating) mirror which makes the rays parallel and directs them toward the grating. The grating disperses the light over a range of angles depending on the wavelength. Rays of a specific wavelength, which depends on the angle between the grating and the incoming rays, travel to the second mirror and are refocused through the exit slit onto the detector. The spectrum is scanned by rotating the grating to sweep light of a particular wavelength onto the detector.

The spectrometer has two detectors. If the dispersed light is allowed to pass through the exit slit as shown in Fig., then it strikes a linear array of 1024 detectors (each one micron wide). The array allows you to simultaneously observe the spectrum over a range of wavelengths, which greatly increases the speed of data taking but at the expense of lower efficiency.

There is a mirror that can be swung into the exiting beam to divert the rays toward a photomultiplier photon detector on the side of the instrument. The phototube can measure only one wavelength at a time but has much lower background noise and is more useful for precision or for low light intensities.

The width of the entrance slits can be individually adjusted. The wider the slits the more light reaches the detector, but on the other hand the resolution is improved by narrowing the slits. Widths of $10 \mu m$ generally give good results. You should use the same setting for the entrance and exit slits.

The entire apparatus is computer controlled. A stepper motor rotates the mirror the grating in adjustable steps (e.g. 0.005 nm/step). The dwell (integration) time at each wavelength is also adjustable (e.g. 2 s/step). During this time the photomultiplier output is integrated (averaged) to improve signal-to-noise. Data analysis options are available for determining peak areas, channel counts, peak wavelengths etc.

The **electromagnet** consists of a soft iron core which taper to a small (~ 1 cm) gap. The core is wound with two coils which generate a field in the gap of about 1.5 T when a current of 10 amperes (max) is applied to the coils. Care must be taken not to overheat the coils which should be allowed to cool whenever they feel moderately warm to the touch.

The magnetic field is measured with a **calibrated Hall probe**. Because of the iron core, the magnet exhibits hysteresis and you cannot count on a given current producing a specific field. You either need to establish a hysteresis curve as described in the appendix or use the Hall for every spectrum you measure. When inserting the Hall probe in the gap be sure to place it at the very center of the gap with the face of the probe parallel to the gap faces.

To measure a spectrum a discharge tube is placed in the gap. Its light is collected from the portion of the tube in the very center of the gap and is

transferred to the spectrometer with a fiber optic feed. A **Polaroid filter** is mounted in the front end of the fiber optic. The orientation of the filter is read from the graduated wheel: 24° corresponds to polarization perpendicular to the field and 114° to parallel.

PROCEDURE: Before starting the experiment, calculate the predicted Zeeman splitting of the 7s6s $^3S_1$ to 6p6s $^3P_0$ transition in mercury -- i.e. calculate g_{eff}. [Combine Eqs. (8) and (10). You will need to use that portion of Table 1 that is appropriate for the transition.] Include this calculation in your final report.

1. Insert the Penray miniature Hg discharge tube into the magnet gap and center the tube. Carefully center the collimating tube for the fiber optic feed on the portion of the Hg tube that is the exact center of the gap. Instructions for starting and operating the spectrometer are given in write-up for the experiment on atomic spectra. Set the Polaroid to 114°. Record the spectrum of the 404.7 nm mercury line. Use a narrow sweep range with long integration time so that the line is spread out with good signal-to-noise. Measure the line width (full width at half maximum intensity) to determine the resolution of the spectrometer. [There will be a residual field of about 0.03 T in the gap, which will produce a small Zeeman splitting. But by setting the Polaroid to parallel polarization, you are eliminating the σ lines and any Zeeman broadening they might contribute to the line.]

2. Turn the magnet and increase the current to 10 amperes. Measure the σ and π spectra (Polaroid at 24° and 114°, respectively) of the 404.7 nm line. Use the Hall probe to measure the magnetic field. For the σ line carefully measure the spectrum for three or four other smaller magnetic fields. [Pay careful attention to the temperature of the magnet coils and let them cool if they become very warm.]

3. Record the zero field and σ and π spectra at 1.5 T of the Hg 435.8 nm and 546.1 nm lines.

4. Replace the Hg tube with the neon discharge tube in the magnet gap. Start the discharge in the neon tube and adjust the rheostat in series with the high voltage transformer to the maximum value which assures reliable starting and discharge operation. The discharge is affected by the applied magnetic field and you will need to adjust the rheostat so that the discharge is stable when the magnetic field power supply is turned up to 5 amperes.

There are a number of nearby lines in the Ne spectrum: 588.2 nm (5), 587.3 nm (5), 587.2 nm (4), 586.8 nm (3), **585.3 nm (10)** , 582.9 nm (2), where the figures in parentheses are relative strengths. Measure the spectrum for the 585.3 nm line for zero magnetic field and for the σ and π polarizations when the magnet current is 5 amps. Repeat for several other magnetic fields. Carefully measure the magnetic field for each spectrum. [You may find it

more convenient to establish a hysteresis curve as described in Appendix 1 and calibrate the field after you have taken your spectra.]

REPORT: The three visible mercury spectral lines that you've studied all represent examples of the anomalous Zeeman effect. In your report calculate the predicted Zeeman spectra for all three cases (Table 1 and Fig. 4 give the results for the blue line).

The 7s6s $^3S_1$ to 6p6s $^3P_0$ transition is least affected by resolution problems. Plot the Zeeman splitting versus B for the σ lines and make a linear fit to determine the experimental value of g_{eff}. Estimate your error and compare with the prediction. Discuss the sources of any discrepancy.

For the 7s6s $^3S_1$ to 6p6s $^3P_1$ transition compare your spectrum for the σ lines with the simulation in Fig. 5. Determine g_{eff} for these lines and compare with the value predicted in the simulation. For the π line prepare a simulation similar to Fig. 5 and compare with your data.

For the 7s6s $^3S_1$ to 6p6s $^3P_2$ transition qualitatively compare your data with the predicted spectrum.

For Neon determine g_{eff} and estimate your error. What can you say about the g-factors of the initial and final states?

REFERENCES:

1. McGraw hill Encyclopedia of Science and Technology, 7th ed, NY 1992, pp 615-617.

2. Halliday, Resnick, and Walker: Fundamentals of Physics, pp 791-792 (derviation of magnetic moment of a circular loop of current)

3. Serway, Moses and Moyer: Modern Physics, pp. 216-226 (normal and anomalous Zeeman effects) (theoretical discussion of Zeeman Effect)

4. G. R. Fowles: Introduction to Modern Optics, p. 247,Figure 8.11 (discussion of polarization characteristics of the Zeeman spectrum)

APPENDIX 1: Establishing *the hysteresis curve* Do not leave the magnet current on for long periods of time above 5 amperes. The magnet will get hot.

Turn on the magnet power supply (be sure to set the output to zero!). Turn on the gaussmeter and calibrate it, following the instructions on top of the instrument

Remove any residual magnetism in the Zeeman magnet ("degauss") as follows: Raise the current to +9 amperes (which current direction is + is arbitrary) and back to zero, switch off the power supply and <u>reverse polarity</u>; raise current to -8 amperes and back to zero, switch off and reverse polarity; raise current to +7 amperes and back to zero, switch off and reverse polarity, etc. Continue until you reach 1/2 ampere.

Hereafter you will maintain a definite magnetic hysteresis curve by always increasing the current up to 8 amperes in the same sense, and by always decreasing the current back down to zero before raising it again. Turn current down to zero (from 8 amperes)before switching off-do not switch off with current flowing. This is good practice with any magnetic circuit, to avoid inductively generated high voltages and possible arcing. Cycle a few times between 0 and 8 amperes (no polarity reversals) to establish your hysteresis loop before beginning observations of Zeeman splitting.

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