Chapter 12
Fluid Mechanics

A fluid is any substance that can flow, such as gases and liquids. In general, gases are compressible, while liquids are difficult to compress. In the first part of this chapter we will investigate **fluid statics**, and in the second half we will investigate **fluid dynamics**–basically, the applications of Newton's 1^{st} law and Newton's 2^{nd} law to fluids.

1 Density

While we've used the concept of *density* before this chapter, we will formally introduce it here. When characterizing the density of a material occupying 3 dimensions, we use the greek symbol "rho" (ρ) which is defined as:

$$\rho = \frac{\text{Mass}}{\text{Volume}} \qquad \text{(definition of density)}$$

The SI units of density are kg/m^3. You will sometimes see two sets of units used when expressing the density of a substance–either g/cm^3, or kg/m^3. See the table in the book for typical densities of some common substances.

Sometimes, the density of material is compared to that of water at 4^o C, and this is called the **specific gravity**. This is really a measure of relative densities, and as you can see from the definition of density, *gravity* has nothing to do with it.

Ex. 5 A uniform lead sphere and a uniform aluminum sphere have the same mass. What is the ratio of the radius of the aluminum sphere to the radius of the lead sphere?

2 Pressure in a Fluid

Another characteristic of a fluid is its *pressure*. The pressure is a scalar quantity the measures the *force/unit area*.

Table 14.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

$$P = \frac{dF_\perp}{dA}$$

The SI units of pressure is the *pascal*, or Pa, and it's equal to 1 $\mathrm{N/m^2}$. The atmospheric pressure due to the weight of the atmosphere (measured at sea level), is defined to be 1 atmosphere, or

$$1 \text{ atmosphere} = 1.013 \times 10^5 \text{ Pa} = 1.013 \text{ bar} = 1013 \text{ millibar} = 14.70 \text{ lb/in}^2$$

2.1 Pressure, Depth, and Pascal's Law

We can derive a general relation between the pressure p at any point in a fluid at rest and the elevation y of the point. We will assume that the density ρ and the acceleration due to gravity g are the same throughout the fluid. If the fluid is at rest, we can apply Newton's 1$^{\text{st}}$ law to determine the pressure at the bottom and top surfaces at their respective elevations y and $y+dy$. Let pA be the force pushing down from the top.

$$\sum F_y = 0 \qquad pA - (p + dp)A + \rho g A \, dy = 0$$

If we divide both side by the area A, as well as dy, we have

$$\frac{dp}{dy} - \rho g = 0 \qquad \text{or} \qquad dp = \rho g \, dy$$

Integrating both sides of this equation we find:

$$p_2 - p_1 = \rho g \, (y_2 - y_1)$$

Sometimes it is convenient to express the pressure p as a function of depth h below the surface. If we do this, we can rewrite the previous equation as

$$p - p_o = \rho g h \qquad p = p_o + \rho g h \tag{1}$$

where h is the depth below the surface of the fluid, and p_o is the pressure at the surface.

2.2 Pascal's Law

Blaise Pascal (1623-1662), a French scientist, recognized that if the pressure at the surface of the fluid increased, the pressure would also be added to all points in the fluid underneath the surface.

Pascal's Law: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

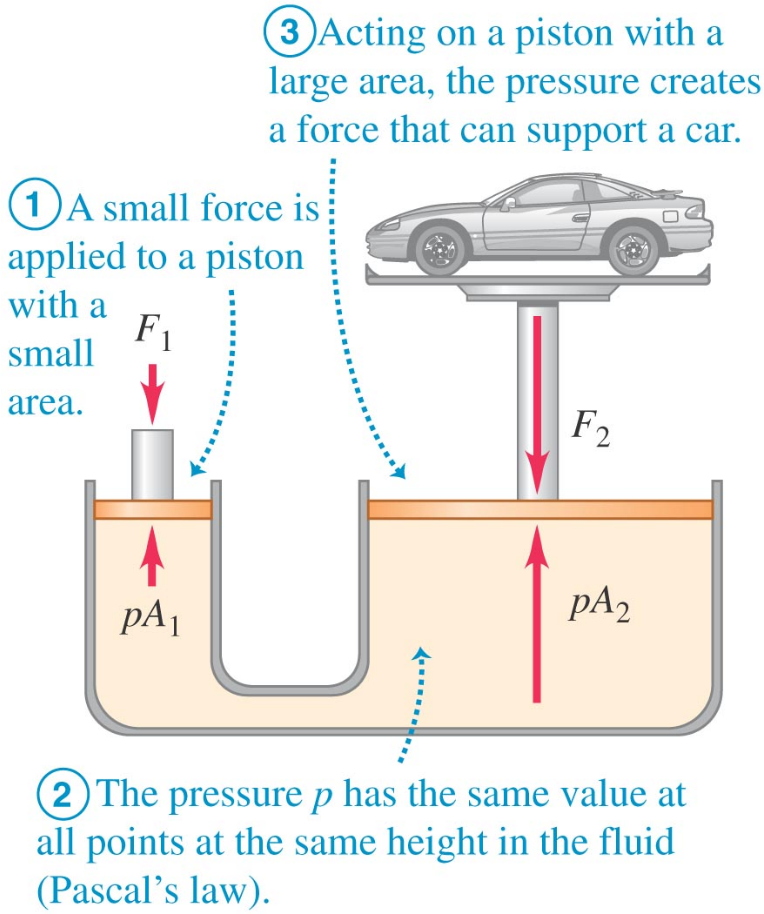
This can be written in the form of an equation. The pressure (and over-pressure) at the same elevations must be the same:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{and} \qquad F_2 = F_1 \frac{A_2}{A_1}$$

The use of Eq. 1 is limited to regions (i.e., depths) where the density does not change appreciably.

2.3 Absolute Pressure, Gauge Pressure, and Pressure Gauges

In most cases, when we use a pressure measuring device, it measures that *gauge pressure*, that is, the pressure relative to the surrounding atmospheric pressure. In

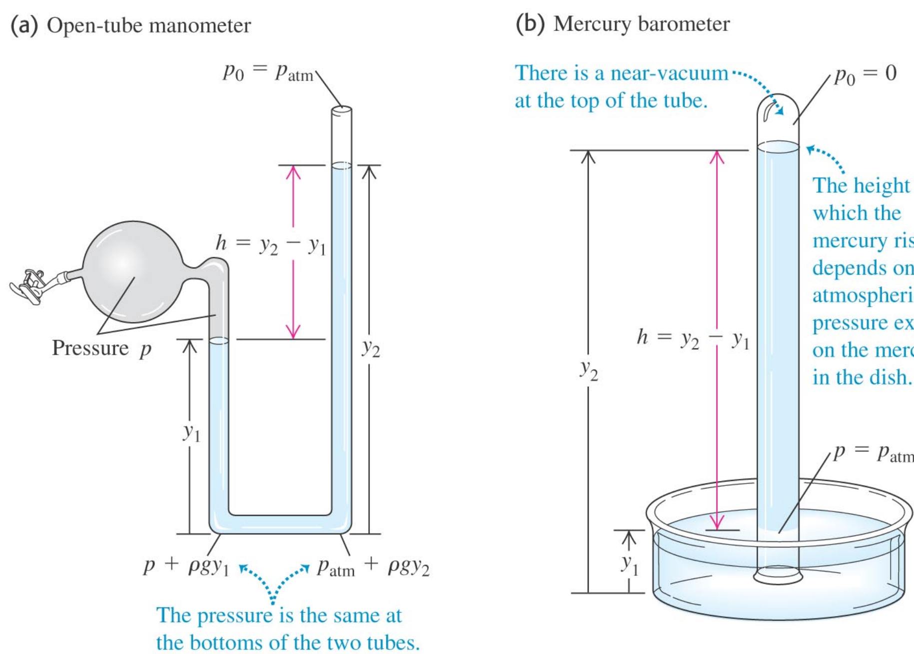


3 Acting on a piston with a large area, the pressure creates a force that can support a car.
1 A small force is applied to a piston with a small area.
F_1
F_2
pA_1
pA_2
2 The pressure p has the same value at all points at the same height in the fluid (Pascal's law).

other words, when you measure the gauge pressure, you're measuring

$$p - p_o = \rho g h \qquad \text{(the gauge pressure)}$$

On the other hand, a mercury barometer measures the absolute pressure, that is, the total *force/unit area* the atmosphere is pushing on us locally.



Ex. 10 A barrel contains a 0.120-m layer of oil floating on water that is 0.250 m deep. The density of the oil is 600 kg/m^3. (a) What is the gauge pressure at the oil-water interface? (b) What is the gauge pressure at the bottom of the barrel?

3 Buoyancy

The *buoyant* force occurs whenever an object is is partially or completely submerged in a fluid while in a gravitational field. The force was first stated by Archimedes:

Archimedes principle: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

The buoyant force is *another* external force that can be used in Newton's laws. The magnitude of the force is

$$F_{\mathrm{B}} = \rho_{\text{fluid}}\, g V_{\text{displ.}} \qquad \text{(the weight of the fluid displaced)}$$

and the direction of the force is upward.

Ex. 17 An electrical short cuts off all power to a submersible diving vehicle when it is 30 m below the surface of the ocean. The crew must push out a hatch of area 0.75 m^2 and weight 300 N on the bottom to escape. If the pressure inside is 1.0 atm, what downward force must the crew exert on the hatch to open it?

Ex. 29 An ore sample weighs 17.50 N in air. When the sample is suspended by a light cord and totally immersed in water, the tension in the cords is 11.20 N. Find the total volume and the density of the sample.

4 Fluid Flow

Starting with this section, we begin to investigate the properties of **fluid dynamics**.

General Concepts of Fluid Flow

Joseph Louis Lagrange (1736-1813) applied particle mechanics to the motion of fluid particles to specify the history of each fluid particle.

Leonhard Euler (1707-1783) took a different approach. He specified the density and the velocity of the fluid at each point in space at each instant of time.

$$\rho(x, y, z, t) \qquad \text{and} \qquad \vec{v}(x, y, z, t)$$

We will focus our attention on what is happening at a particular point in space at a particular time, rather than on what is happening to particular fluid particle.

What are some of the general characteristics of fluid flow?

1. *Fluid flow can be steady or nonsteady.* Fluid flow is *steady* if the pressure, density, and flow velocity are constant in time at every point of the fluid

2. *Fluid flow can be compressible or incompressible.* Fluid flow is *incompressible* if the density ρ is a constant, independent of x, y, z, and t.

3. *Fluid flow can be viscous or nonviscous.* Viscosity in fluid motion is analogous to the friction in the motion of solids–kinetic energy is transformed into internal energy by *viscous* forces. The greater the *viscosity*, the greater the external force or pressure must be applied to maintain the flow.

We will mostly consider the motion of **ideal fluids** which can be regarded as steady, incompressible, and nonviscous.

Streamlines and the Equation of Continuity

In steady flow, the velocity $\vec{v}$ at a given point P is constant in time. Every particle moving through P follows the same path, called a *streamline*.

Every fluid particle that passes through P subsequently passes through other points Q and R. Likewise, every fluid particle that passes through R must have previously passed through points Q and P. Connecting these points results in the formation of a *streamline*.

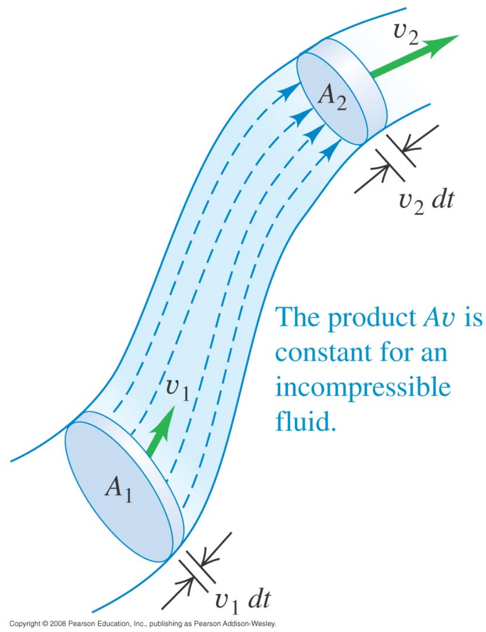
The velocity vector $\vec{v}$ can change throughout the streamline, however, it is constant and *tangent* at a particular point along the streamline.

Let's consider the flow of a fluid through a *tube of flow* entering at area A_1 and exiting at a point area A_2. The fluid particles enter A_1 with a velocity v_1 and exit A_2 with a velocity v_2.

The amount of mass entering A_1 in a time δt is:

$$\delta m_1 = \rho_1 A_1 v_1 \delta t$$

We define the *mass flux* as the mass of fluid per unit time passing through any



cross section:

$$\frac{\delta m}{\delta t} = \rho_1 v_1 A_1 \qquad \text{(the mass flux at } A_1\text{)}$$

Likewise, this must also be the *mass flux* at A_2 (i.e., conservation of mass):

$$\frac{\delta m}{\delta t} = \rho_2 v_2 A_2 \qquad \text{(the mass flux at } A_2\text{)}$$

Thus we have:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \qquad \text{(Equation of continuity–compressible fluid)}$$

where $\rho A v =$ constant.

We can take this one step further if we consider incompressible fluids where $\rho_1 = \rho_2$. In this case, we have:

$$A_1 v_1 = A_2 v_2 \qquad \text{(Equation of continuity–incompressible fluid)}$$

where $R = Av$ is devined as the *volume flow rate*.

Ex. 41 Water is flowing in a pipe with a circular cross section but with varying cross-sectional area, and at all points the water completely fills the pipe. (a) At one point in the pipe the radius is 0.150 m. What is the speed of the water at this point if water is flowing into this pipe at a steady rate of 1.20 m^3/s? (b) At a second point in the pipe the water speed is 3.80 m/s. What is the radius of the pipe at this point?

5 Bernoulli's Equation

The flow of an ideal fluid through a pipe or a tube is influenced by the following conditions:

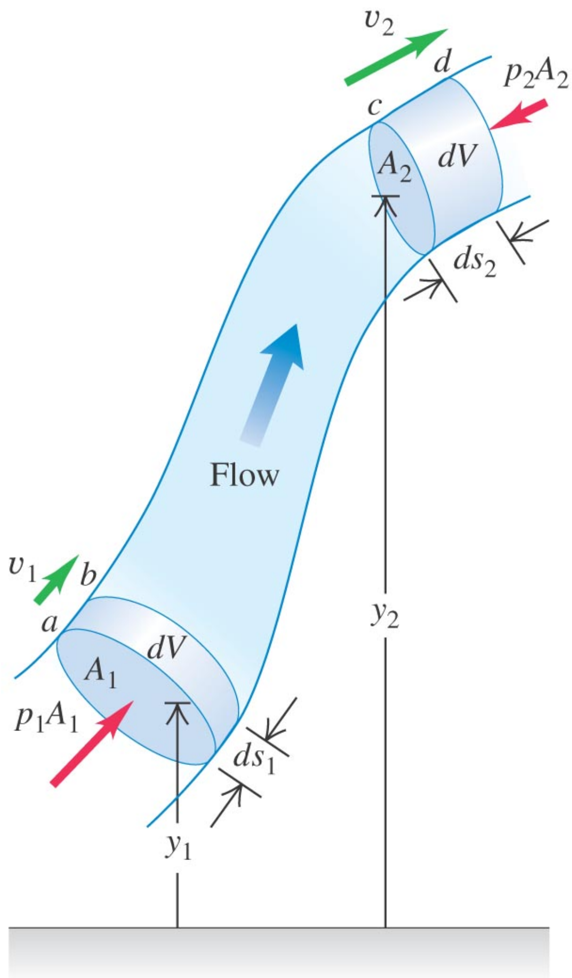
1. the cross-sectional area of the pipe may change,
2. the inlet and outlet of the pipe may be at different elevations, and
3. the inlet and outlet pressures may be different.

The work-energy theorem is used to develop Bernoulli's equation.

$$W_{ext} \;=\; \Delta K \qquad \text{where} \qquad W_{ext} = W_1 + W_2 + W_{\text{grav}}$$

$$\begin{aligned} W_1 &= \; p_1 A_1 \, ds_1 \\ W_2 &= -\; p_2 A_2 \, ds_2 \\ W_{\text{grav}} &= -\; \rho \, dV \, g(y_2 - y_1) \end{aligned}$$

where $A_1 \, ds_1 \;=\; A_2 \, ds_2 \;=\; dV \;=\; dm/\rho$



$$\Delta K \;=\; \frac{1}{2}\,dm\,v_2^2 \;-\; \frac{1}{2}\,dm\,v_1^2$$

Rewriting the work-energy equation, we have:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 \;=\; p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \qquad \text{(Bernoulli's Equation)}$$

1. *Static pressure* is a special case of fluid dynamics. Setting $v_1 = v_2 = 0$, we have:

$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

2. *Dynamic pressure* for a fluid flowing horizontally (no change in potential energy)

$$p_2 - p_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

Bernoulli's equation is basically an equation that describes the conservation of energy density, namely:

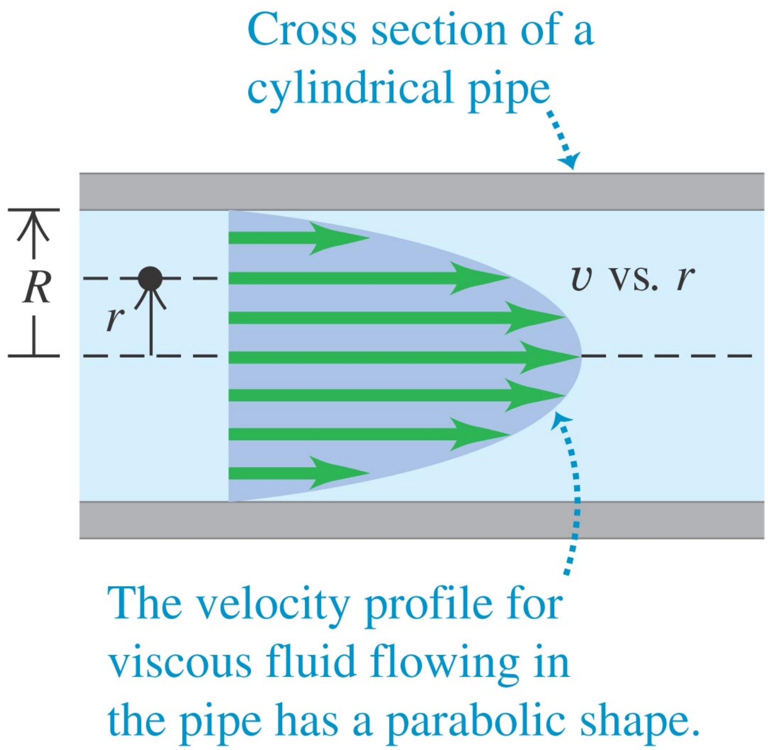
$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

Ex. 48 A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find a) the speed of efflux; b) the volume discharged per unit time.

Ex. 60 **Ballooning on Mars.** It has been proposed that we could explore Mars using inflated balloons to hover just above the surface. The buoyancy of the atmosphere would keep the balloon aloft. The density of the Martian atmosphere is 0.0154 $\mathrm{kg/m^3}$ (although this varies with temperature). Suppose we construct these balloons of a thin but tough plastic having a density such that each square meter has a mass of 5.00 g. We inflate them with a very light gas whose mass we can ignore. (a) What should be the radius and mass of these balloons so they just hover above the surface of Mars? (b) If we released one of the balloons from part (a) on earth, where the atmospheric density is 1.20 $\mathrm{kg/m^2}$, what would be its initial acceleration assuming it was the same size as on Mars? Would it go up or down? (c) If on Mars these balloons have five times the radius found in part (a), how heavy an instrument package could they carry?

6 Viscosity and Turbulence

There are no equations in this section, but it's worth reading–especially the part describing the velocity profile for a fluid moving through a pipe.



Viscosity
The pressure difference required to sustain a given volume flow rate through a cylindrical pipe of length L and radius R is proportional to L/R^4. If the radius R is decreased by 10%, this increases the required pressure difference by a factor $(1/0.90)^4 = 1.52$, or a 52% increase.

Superfluid
A superfluid is a fluid that displays zero viscosity. An example of this is superfluid helium at temperatures below 2.17 K. If you stir superfluid helium and set it into rotational motion, it will create a vortex whose kinetic energy does not dissipate. In other words, it just keeps rotating.

Prob. xx This is a Venturi meter (take a look at Example 12.9 for more details). The horizontal pipe shown below has a cross-sectional area of 40.0 cm^2 at the wider portions and 10.0 cm^2 at the constriction. Water is flowing in the pipe, and the discharge from the pipe is 6.00×10^{-3} m^3/s (6.00 L/s). Find (a) the flow speeds at the wide and narrow portions: (b) the pressure difference between these portions; (c) the difference in height between the mercury columns in the U-shaped tube.

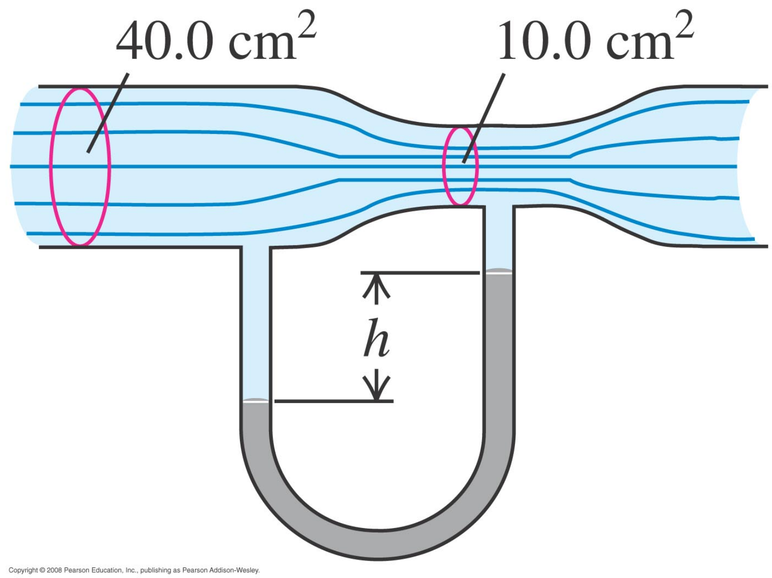


Figure 1: Example of a Venturi meter