Chapter 13 Solutions In-Class

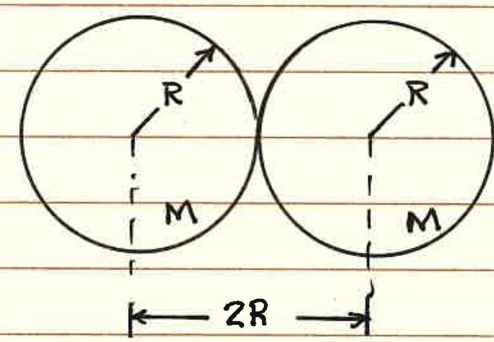
Ex. 1 What is the ratio of the gravitational pull of the sun on the moon to that of the earth on the moon?

$$\frac{F_{sun/moon}}{F_{earth/moon}} = \frac{GM_s m / d_{sun}^2}{GM_E m / d_{earth}^2} = \frac{M_s}{M_e}\left(\frac{d_{earth}}{d_{sun}}\right)^2$$

$$\frac{F_{sun/moon}}{F_{earth/moon}} = \frac{1.99\times10^{30}\ kg}{5.97\times10^{24}\ kg}\left(\frac{3.84\times10^{8}\ m}{1.50\times10^{11}\ m}\right)^2 = 2.18$$

Ex. 4 Two uniform spheres, each with mass M and radius R, touch one another.

$$F = G\frac{MM}{(2R)^2} = \frac{GM^2}{4R^2}$$



Ex. 14 Rhea, one of Saturn's moons, has a radius of 765 km and an acceleration due to gravity of 0.278 m/s^2 at its surface.

$M = ?$ $\rho = ?$

$$a = \frac{GM}{R^2} \qquad M = \frac{aR^2}{G} = \frac{0.278\ m/s^2\ (765\times10^3\ m)^2}{6.67\times10^{-11}\ N\cdot m^2/kg^2}$$

$$\boxed{M = 2.44\times10^{21}\ kg}$$

$$\rho = \text{density} \equiv \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2.44\times10^{21}\ kg}{4/3\,\pi\,(765\times10^3\ m)^3}$$

$$\boxed{\rho = 1.30\times10^{3}\ kg/m^3}$$

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Ex. 10 A planet orbiting a distant star has a radius $3.24\times10^{6}\,m$.

$V_{esc} = 7.65\times10^{3}\,m/s = \sqrt{\frac{GM\,2}{R}}$ $\quad V_{esc}^{2} = 2R\left(\frac{GM}{R^2}\right)$

g = acceleration due to gravity $= \frac{GM}{R^2}$. So, $V_{esc}^{2} = 2Rg$

$$g = \frac{V_{esc}^{2}}{2R} = \frac{(7.65\times10^{3}\,m/s)^2}{2(3.24\times10^{6}\,m)} = \boxed{9.03\ m/s^2}$$

Ex. 25 For a satellite to be in a circular orbit 890 km above the surface of the earth, ...

a.) $V_{orb} = ?$ $\quad V_{orb} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_E}{R_E+h}} = \sqrt{\frac{6.67\times10^{-11}\ 5.98\times10^{24}}{6.37\times10^{6}+890\times10^{3}}}$

$$\boxed{V_{orb} = 7412\ m/s}$$

b.) $T = ?$ $\quad 2\pi r = V_{orb}\,T$ $\quad T = \frac{2\pi(R_E+h)}{V_{orb}} = \frac{2\pi(6.37\times10^{6}+890\times10^{3})}{7412\ m/s}$

$$T = 6154\,s\left(\frac{1\ hr}{3600\,s}\right) = \boxed{1.71\ hours}$$

Ex. 30 Hot Jupiters. In 2004 astronomers reported the discovery of a large Jupiter-sized planet

$r = (1/9)\,r_{Mercury} = \frac{1}{9}(5.79\times10^{10}\,m) = 6.43\times10^{9}\,m$

$T = 3.09\ days\left(\frac{86{,}400\,s}{1\ day}\right) = 2.67\times10^{5}\,sec.$ $\quad M = M_{star} = ?$

$T^2 = \frac{4\pi^2 r^3}{GM}$ $\quad M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(6.43\times10^{9})^3}{(6.67\times10^{-11})(2.67\times10^{5})^2}$

$$\boxed{M = 2.21\times10^{30}\ kg}$$

b.) $v = ?$ $\quad v = \frac{2\pi r}{T} = \frac{2\pi(6.43\times10^{9})}{2.67\times10^{5}}$ $\quad \boxed{v = 151.4\ km/s}$

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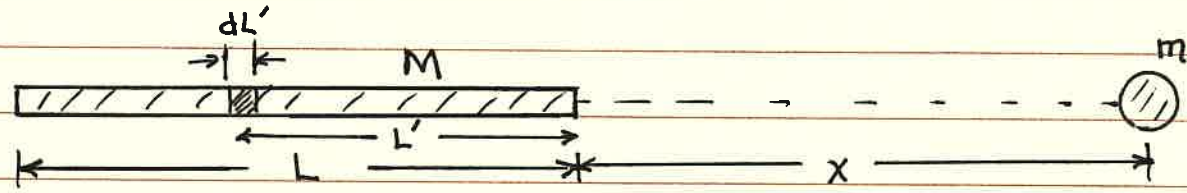
Ex. 38 A thin, uniform rod has length L and mass M. A small uniform sphere of mass m is placed a distance x from one end of the rod .... $U_{gr} = ?$



The solution for this problem can be found in my lecture notes.

$$dU = -\frac{G\,m\,\lambda\, dL'}{x+L'}$$

$$U_{gr} = -G\,m\,\lambda \int_0^L \frac{dL'}{x+L'}$$

$$U_{gr} = -Gm\lambda \ln\left(\frac{L+x}{x}\right) = -Gm\lambda \ln\left(1+\frac{L}{x}\right)$$

From Appendix C $\Rightarrow \ln(1+z) = z - \frac{z^2}{2} + \cdots$

$$U_{gr} = -Gm\lambda\left(\frac{L}{x} - \frac{L^2}{2x^2} + \cdots\right) = -\frac{Gm\overset{M}{(\lambda L)}}{x}\left(1 - \frac{L}{2x} + \cdots\right)$$

$$U_{gr} \cong -\frac{GMm}{x}\left(1 - \frac{L}{2x}\right)$$

For $x \gg L$ $\quad U_{gr} \to -\frac{GMm}{x}$

$$F_{gr} = -\frac{dU}{dx} = -\frac{d}{dx}\left(-Gm\lambda \ln\left(1+\frac{L}{x}\right)\right) = +Gm\lambda \frac{\left(-L/x^2\right)}{1+L/x}$$

$$\boxed{F_{gr} = -\frac{Gm\overbrace{(\lambda L)}^{M}}{x^2 + Lx}}$$

$$F_{gr}(x) = -\frac{GMm}{x^2}\,\frac{1}{1+L/x}$$

As $x \gg L$, then $\frac{1}{1+L/x} \to 1$, then $\boxed{F_{gr}(x) \cong -\frac{GMm}{x^2}}$

... as expected for a point-like object.

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Ex. 43 At the Galaxy's Core. Astronomers have observed a small massive object at the center of our Milky Way galaxy.

Diameter = 15 light-years $= 15\,(3 \times 10^{8}\,\text{m/s})(3.16 \times 10^{7}\,\text{s}) = 1.42 \times 10^{17}\,\text{m}$

velocity = 200 km/s $= 200 \times 10^{3}\,\text{m/s}$

$M = ?$

V = velocity of a circular orbit $= \sqrt{\dfrac{GM}{r}}$ $\qquad V^2 = \dfrac{GM}{r}$

$$M = \frac{rV^2}{G} = \frac{(7.10 \times 10^{16}\,\text{m})(200 \times 10^{3}\,\text{m/s})^2}{6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = 4.26 \times 10^{37}\,\text{kg}$$

$$M = 4.26 \times 10^{37}\,\text{kg}\left(\frac{1\,M_\odot\ (\text{solar mass})}{1.99 \times 10^{30}\,\text{kg}}\right) = \boxed{2.14 \times 10^{7}\,M_\odot}$$

b.) No, it cannot be a single ordinary star. It's mass is HUGE.

c.) R_s = Schwarzschild radius $= \dfrac{2GM}{c^2} = \dfrac{2\,(6.67 \times 10^{-11})\,4.26 \times 10^{37}}{(3.00 \times 10^{8})^2}$

$\boxed{R_s = 6.31 \times 10^{10}\,\text{m}}$ Yes. It will fit inside the earth's orbit around the sun. $1.50 \times 10^{11}\,\text{m}$

Prob. 56 Your starship, the Aimless Wanderer, lands on the mysterious planet Mongo. A 2.00 kg stone $V_{oy} = 12.0\,\text{m/s}$ $\quad t = 4.80\,\text{s}$

$y = 0 \qquad M = ?$

Eq. 3 $\Rightarrow$ $\overset{=0}{y} = V_{oy}t - \frac{1}{2}\, g_{Mongo}\, t^2$ $\qquad g_{Mongo} = \dfrac{2V_{oy}}{t} = \dfrac{2\,(12.0\,\text{m/s})}{4.80\,\text{s}}$

$g_{Mongo} = 5.00\,\text{m/s}^2$ $\qquad$ a.) $M = ?$ $\qquad g_{Mongo} = \dfrac{GM_{Mongo}}{R^2_{Mongo}} = \dfrac{GM_{Mongo}}{(C_{Mongo}/2\pi)^2}$

Circumf.

$$M_{Mongo} = \frac{C^2_{Mongo}\, g_{Mongo}}{(2\pi)^2 G} = \frac{(2.00 \times 10^{8}\,\text{m})^2 (5.00\,\text{m/s}^2)}{4\pi^2 (6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)} = \boxed{7.60 \times 10^{25}\,\text{kg}}$$

Chapter 13 Solutions In-Class

Prob. 56 cont'd

b.) Aimless Wanderer goes into circular orbit 30,000 km above the surface, calculate the period of its orbit.

$$R_{Mongo} = \text{Circ.}/2\pi = 2\times10^8\,\text{m}/2\pi = 3.18\times10^7\,\text{m}$$

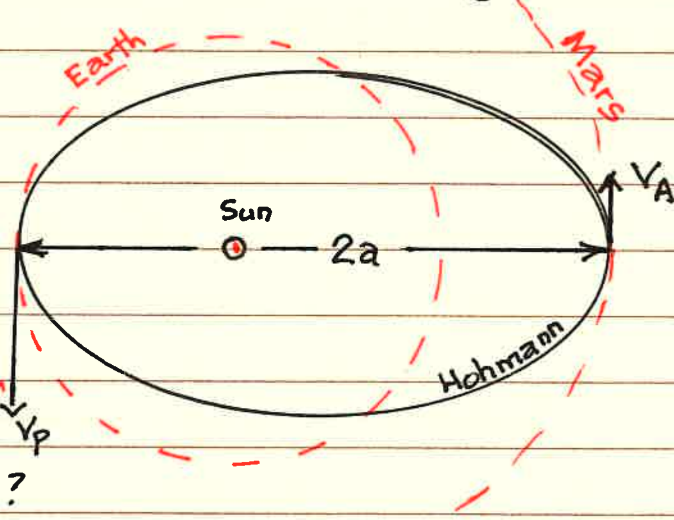
$$T = \frac{2\pi}{\sqrt{GM_{Mongo}}}\, r^{3/2} = \frac{2\pi}{\sqrt{GM_{Mongo}}}\,(R_M + h)^{3/2}$$

$$T = \frac{2\pi\,(3.18\times10^7\,\text{m} + 3.00\times10^7\,\text{m})^{3/2}}{\sqrt{(6.67\times10^{-11})(7.60\times10^{25})}} \qquad T = 42{,}874\,\text{s}\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)$$

$$\boxed{T = 11.9 \text{ hours}}$$

Prob. 79

Interplanetary Navigation: The most efficient way to send a spacecraft from the earth to another planet is by using a Hohmann Transfer Orbit.



Earth
$$V_E = \sqrt{\frac{GM_\odot}{d_E}} = 2.98\times10^4\,\text{m/s}$$

Mars
$$V_M = \sqrt{\frac{GM_\odot}{d_M}} = 2.41\times10^4\,\text{m/s}$$

What are the values for V_A and V_P for the elliptical Hohmann transfer orbit?
Aphelion Perihelion

ellipse Sun satellite = spacecraft

$$E = \frac{-GM_\odot m}{2a} = \frac{-GM_\odot m}{d_E + d_M} = KE(d_E) + PE(d_E)$$

$$\frac{-GM_\odot \cancel{m}}{d_E + d_M} = \frac{1}{2}\cancel{m}V_P^2 - \frac{GM_\odot \cancel{m}}{d_E} \qquad V_P^2 = 2GM_\odot\left(\frac{1}{d_E} - \frac{1}{d_E + d_M}\right)$$

Chapter 13 In-Class Solutions

Prob. 79 cont'd

$$v_P = \sqrt{2(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{1}{1.50\times10^{11}} - \frac{1}{(1.50+2.28)\times10^{11}}\right)}$$

$$\boxed{v_P = 3.27\times10^4\ \text{m/s}} \quad > \quad \overset{\text{Earth}}{v_E} = 2.98\times10^4\ \text{m/s}$$

Sun   Satellite = spacecraft

$$v_A = ? \qquad E_{ellipse} = -\frac{GM_\odot \cancel{m}}{d_E + d_M} = \frac{1}{2}\cancel{m}v_A^2 - \frac{GM_\odot \cancel{m}}{d_M}$$

$$v_A^2 = 2GM_\odot\left(\frac{1}{d_M} - \frac{1}{d_M + d_E}\right)$$

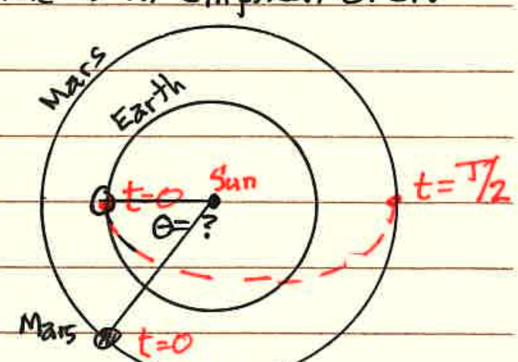
$$v_A = \sqrt{2(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{1}{2.28\times10^{11}} - \frac{1}{(1.50+2.28)\times10^{11}}\right)}$$

$$\boxed{v_A = 2.15\times10^4\ \text{m/s}} \quad < \quad \overset{\text{Mars}}{v_M} = 2.41\times10^4\ \text{m/s}$$

Earth to Mars: You must increase the speed of the spacecraft in the direction of the earth's orbit (motion)

Mars to Earth: You must decrease the speed of the spacecraft with respect to Mars' orbital vebcity.

$$T = \frac{2\pi}{\sqrt{GM_\odot}}\, a^{3/2}$$ is the period of the full elliptical orbit



However, we only want to know T/2, half the period.

The time-of-flight from Earth → Mars.

Chapter 13 In-Class Solutions

Prob. 79 cont'd

$$\frac{T}{2} = \frac{\pi}{\sqrt{GM_\odot}} a^{3/2} \quad \text{where } a = \frac{d_E + d_M}{2} = 1.89 \times 10^{11}\,\text{m}$$

$$\frac{T}{2} = \frac{\pi\,(1.89\times10^{11})^{3/2}}{\sqrt{(6.67\times10^{-11})(1.99\times10^{30})}} = 2.24\times10^{7}\,\text{s} = \boxed{259 \text{ days}}$$

c.) Where should mars be with respect to the earth when launching from earth → mars ?

$$T_{Mars} = \text{period for mars' orbit} = \frac{2\pi}{\sqrt{GM_\odot}} a^{3/2} \quad \text{where } a = d_M$$

$$T_{Mars} = \frac{2\pi\,(2.28\times10^{11})^{3/2}}{\sqrt{(6.67\times10^{-11})(1.99\times10^{30})}} = 5.94\times10^{7}\,\text{s} = \underline{\underline{688 \text{ days}}}$$

Hohmann transfer

$$\text{fraction of mars' orbit} = \frac{T/2}{T_{mars}} = \frac{259}{688} = 0.376$$

$$0.376\,(360^\circ) = 136^\circ \text{ in 259 days}$$

See the figure on the previous page for θ

$$\boxed{\theta = 180^\circ - 136^\circ = 44^\circ}$$

Mars must be ahead of the earth by 44° at launch time if it is going to be at the aphelion of the Hohmann transfer orbit.