In-Class Homework Problems Ch. 14 Periodic Motion

Ex. 3 The tip of a tuning fork goes through 440 oscillations in 0.500 sec.

Find ω, T. $\omega = 2\pi f = 2\pi \underbrace{\left(\frac{440 \text{ vibrations}}{0.500 \text{ sec}}\right)}_{f = 800 \text{ Hz}}$

$\boxed{\omega = 5.53 \times 10^{3} \text{ rad/s}}$

$\omega = 2\pi f = 2\pi/T \Rightarrow T = \frac{1}{f} = \frac{1}{800 \text{ Hz}} = 1.25 \times 10^{-3} \text{ s}$

$\boxed{T = 1.25 \times 10^{-3} \text{ s}}$ or 1.25 ms (milliseconds)

Ex. 9 When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, ... $f = 6.00$ Hz

a.) Find the period, $T = \frac{1}{f} = \frac{1}{6.00 \text{ Hz}} \Rightarrow \boxed{T = 0.16\overline{66} \text{ sec}}$

b.) $\omega = ?$ $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{0.1\overline{66} \text{ sec.}}$ $\boxed{\omega = 37.7 \frac{\text{rad}}{\text{s}}}$

c.) m = mass = ? $\omega^2 = \frac{k}{m}$ $m = \frac{k}{\omega^2} = \frac{120 \text{ N/m}}{(37.7 \text{ rad/s})^2}$

Ex. 11 ... 2 pages later

$\boxed{m = 8.44 \times 10^{-2} \text{ kg}}$ or $m = 84.4$ grams

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Ex. 28 A harmonic oscillator has angular frequency ω and amplitude A.

a.) What are "x" and "v" when KE = PE (i.e., K = U)

$K = U = \frac{1}{4}kA^2 = \frac{1}{4}m\omega^2 A^2$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

"v" → $\frac{1}{2}mv^2 = \frac{1}{4}\underbrace{m\omega^2}_{k} A^2$ $v^2 = \frac{1}{2}\omega^2 A^2$ $\boxed{v = \frac{\omega A}{\sqrt{2}}}$

"x" → $\frac{1}{2}kx^2 = \frac{1}{4}\underbrace{m\omega^2}_{k} A^2$ $x^2 = \frac{1}{2}A^2$ $\boxed{x = \frac{A}{\sqrt{2}}}$

Magnitude Only

Ex. 28 cont'd

b.) How often does this occur in each cycle?

Let's assume that the displacement $x(t) = A\cos(\omega t)$

$t_1 \rightarrow \frac{A}{\sqrt{2}} = A\cos(\omega t) \rightarrow \cos(\omega t) = \frac{1}{\sqrt{2}} \quad \omega t = \cos^{-1}(1/\sqrt{2}) = \frac{\pi}{4}$

$\boxed{t_1 = \pi/4\omega}$

$t_2 \rightarrow -\frac{A}{\sqrt{2}} = A\cos(\omega t) \rightarrow \cos(\omega t) = -\frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

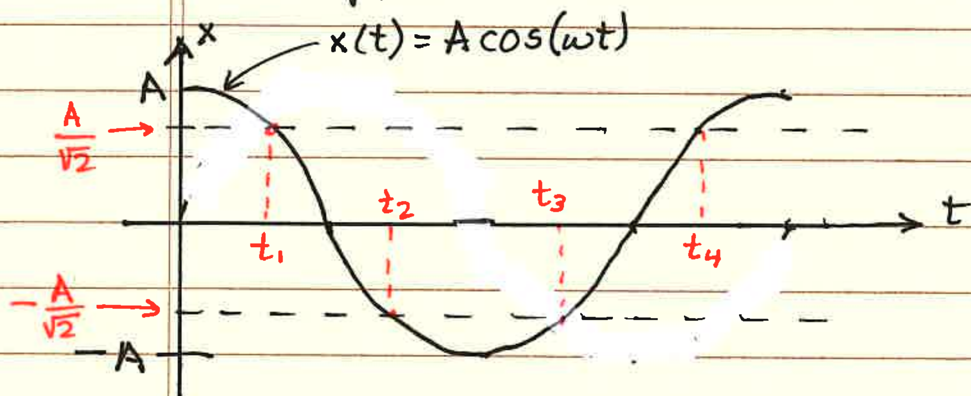
$\boxed{t_2 = \frac{3\pi}{4\omega}}$

$t_3 \rightarrow -\frac{A}{\sqrt{2}} = A\cos(\omega t) \rightarrow \cos(\omega t) = -\frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{5\pi}{4}$

$\boxed{t_3 = \frac{5\pi}{4\omega}}$

$t_4 \rightarrow \frac{A}{\sqrt{2}} = A\cos(\omega t) \rightarrow \cos(\omega t) = +\frac{1}{\sqrt{2}} \rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{7\pi}{4}$

$\boxed{t_4 = \frac{7\pi}{4\omega}}$



What is the spacing of these time intervals?

They are all the same $\Delta t = \frac{2\pi}{4\omega}$ $\boxed{\Delta t = \frac{\pi}{2\omega} \text{ (seconds)}}$

Note: $\omega = \frac{2\pi}{T}$ So, $\boxed{\Delta t = \frac{\pi}{2 \times} \left(\frac{T}{2\pi}\right) = \frac{T}{4}}$ one-quarter of the full period.

c.) At $x = \frac{A}{2}$ $\quad U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A^2}{4}\right) = \frac{1}{8}kA^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{1}{4}E$

$K = E - U = \frac{3}{4}E$

$\boxed{U = \frac{1}{4}E \text{ and } K = \frac{3}{4}E}$

DATE	
TOPIC	

Ch. 14 Periodic Motion

Ex. 11 An object is undergoing SHM with a period of 0.900 s and amplitude 0.320 m.

a.) At $t=0$ $x = 0.320$ m and $v_x = 0$

What is Δt for $x \to 0.320$ m to $x = 0.160$ m.

$$\Delta t = (t - 0) \text{ sec.}$$

In general $x(t) = A\cos(\omega t + \phi)$

In our case, $\phi = 0$ because $x(0) = A$

So, $x(t_1) = A\cos(\omega t_1) \to 0.160\text{ m} = (0.320\text{ m})\cos(\omega t)$

$\cos(\omega t) = \frac{1}{2}$ $\omega t = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\boxed{t_1 = \frac{\pi}{3\omega}}$

$$t_1 = \frac{\pi}{3}\frac{T}{2\pi} = \frac{T}{6} = \frac{0.900\text{ s}}{6} = \boxed{0.150\text{ sec}}$$

b.) What is Δt for $x \to 0.160$ m to 0.00 m.

$x(t_2) = A\cos(\omega t_2) \Rightarrow 0.00\text{ m} = (0.320\text{ m})\cos(\omega t_2)$

$\Rightarrow \cos(\omega t_2) = 0$ $\Rightarrow \omega t_2 = \frac{\pi}{2}$ radians

$$t_2 = \frac{\pi}{2\omega} = \frac{\pi}{2}\frac{T}{2\pi} = \frac{T}{4} = \frac{0.900\text{ s}}{4} = 0.225\text{ s}$$

$$\Delta t = t_2 - t_1 = 0.225\text{ s} - 0.150\text{ s}$$ $\boxed{\Delta t = 0.075\text{ sec}}$

DATE

TOPIC (3)

In-Class Homework Problems Ch. 14 Periodic Motion

Ex. 36 A proud deep-sea fisherman hangs a 65.0 kg fish from an ideal spring....

$\Delta x = 0.180\ m$

a.) $k = ?$ $F = kx$ "force acting on the spring"

$$mg = kx \qquad k = \frac{mg}{x} = \frac{(65.0\ kg)(9.8\ m/s^2)}{0.180\ m} = 3.54 \times 10^3\ \frac{N}{m}$$

$$\boxed{k = 3.54 \times 10^3\ N/m}$$

b.) $T = ?$ Period of oscillation $\omega = \sqrt{k/m}$ $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{65.0\ kg}{3.54\times10^3\ N/m}} \qquad \boxed{T = 0.851\ sec}$$

c.) $V_{max} = ?$ $V_{max} = A\omega = A\left(\frac{2\pi}{T}\right) = 0.05\,m\left(\frac{2\pi}{0.851 s}\right)$

$$\boxed{V_{max} = 0.369\ m/s}$$

Ex. 46 A pendulum on mars ... $T_{earth} = 1.60$ sec.

What is its period on the surface of mars? $g_{mars} = 3.71\ \frac{m}{s^2}$

$$\omega = \sqrt{\frac{g}{L}} \rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \frac{T_{mars}}{T_{earth}} = \frac{2\pi\sqrt{L/g_{mars}}}{2\pi\sqrt{L/g_{earth}}} = \sqrt{\frac{g_{earth}}{g_{mars}}}$$

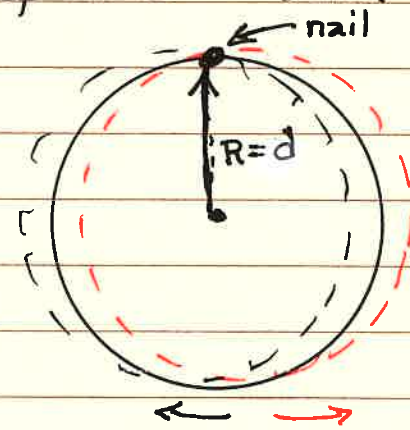
$$T_{mars} = T_{earth}\sqrt{\frac{g_{earth}}{g_{mars}}} = 1.60 s\sqrt{\frac{9.80}{3.71}} = 2.60\ sec$$

$$\boxed{T_{mars} = 2.60\ sec.}$$

In-Class Homework Problems Ch. 14

Ex. 50 We want to support a thin hoop on a horizontal nail.

$T = 2.0$ seconds $\quad R = ?$



$$\omega = \sqrt{\frac{Mgd}{I}}$$

$d = R$

$$I = I_{cm} + Md^2 = MR^2 + MR^2$$

$$I = 2MR^2$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{MgR}{2MR^2}} = \sqrt{\frac{g}{2R}}$$

$$\frac{4\pi^2}{T^2} = \frac{g}{2R} \Rightarrow R = \frac{gT^2}{8\pi^2} = \frac{(9.8\,m/s^2)(2.0\,s)^2}{8\pi^2}$$

$$\boxed{R = 0.496\,m}$$

Ex. 57 An unhappy 0.300 kg rodent, moving on the end of a spring... $k = 2.50$ N/m with a damping force $F_x = -bv_x$

a.) $b = 0.900$ kg/s find f.

$$\omega' = 2\pi f' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \Rightarrow f' = \frac{1}{2\pi}\sqrt{\frac{2.50}{0.300} - \frac{(0.90)^2}{4(0.30)^2}}$$

$$\boxed{f' = 0.393\,Hz}$$

b.) $b = ?$ for critically damped. $\quad b = 2\sqrt{km}$

$$b = 2\sqrt{(2.50\,N/m)(0.300)} = 1.73\,kg/s \qquad \boxed{b = 1.73\,kg/s}$$