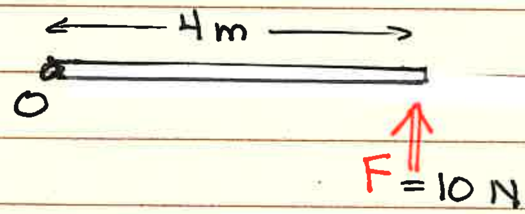
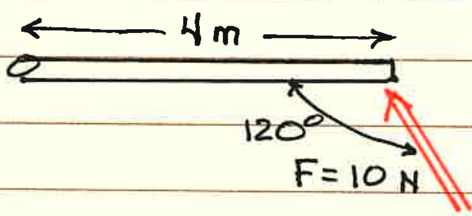
DATE | TOPIC ①

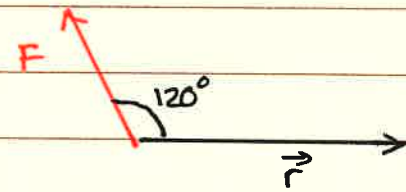
Chapter 10 In-Class Solutions

Ex. 1 Calculate the torque (magnitude and direction) about point O,

a.) 

$$\tau = rF\sin 90^\circ$$
$$= 4\,\text{m}\,(10\,\text{N})\,1$$
$$\boxed{\tau = 40\ \text{N}\cdot\text{m} \quad (+)\ \text{torque}}$$

b.) 

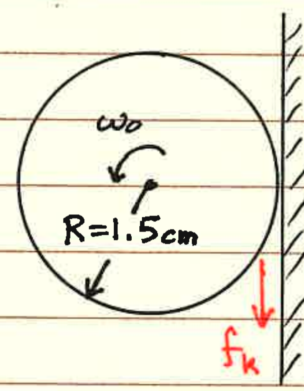


$$\tau = rF\sin(120^\circ) = 4\,\text{m}\,(10\,\text{N})\sin 120^\circ$$

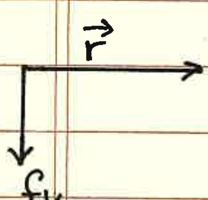
$$\boxed{\tau = 34.64\ \text{N}\cdot\text{m} \qquad (+)\ \text{torque}}$$

And so on .... (c) (d) (e) (f)

Ex. 11 A machine part has the shape of a solid uniform sphere of mass 225 g and a diameter 3.00 cm. $f_k = 0.0200$ N



$$\tau = I\alpha \qquad \vec{\tau} = \vec{r} \times \vec{F} \quad (-)$$



$$= (1.5\times10^{-2}\,\text{m})(0.0200\,\text{N})$$
$$\tau = -3.00\times10^{-4}\ \text{N}\cdot\text{m}$$

$$\alpha = \frac{\tau}{I} = \frac{-RF}{\frac{2}{5}MR^2} = -\frac{5}{2}\frac{F}{MR}$$

$$\alpha = -\frac{5}{2}\frac{(0.0200\,\text{N})}{(0.225\,\text{kg})(0.015\,\text{m})} \qquad \boxed{\alpha = -14.8\ \text{rad/s}^2}$$

b.) $\Delta\omega = -22.5$ rad/s $\quad \Delta\omega = \alpha\Delta t$ (Eq. 1) $\quad \Delta t = \Delta\omega/\alpha$

$$\Delta t = -22.5\ \text{rad/s} \,/\, (-14.8\ \text{rad/s}^2) \qquad \boxed{\Delta t = 1.52\ \text{sec.}}$$

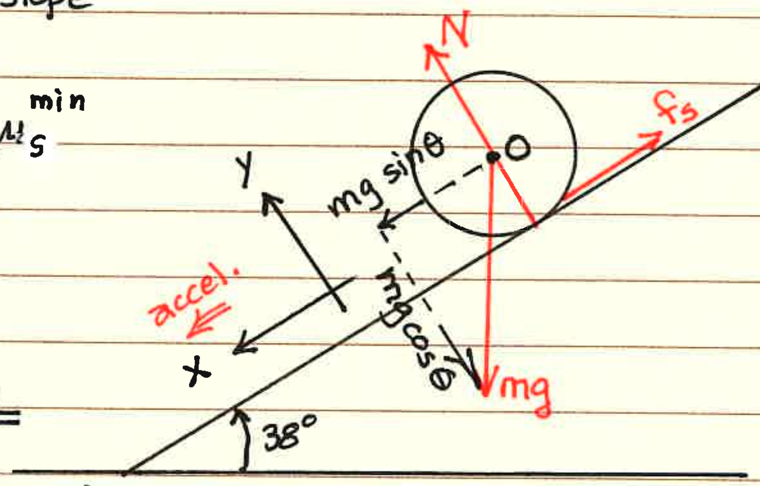
DATE

TOPIC ②

Chapter 10 In-Class Solutions

Ex. 24 A hollow spherical shell with a mass 2.00 kg rolls without slipping down a 38° slope

a.) Find a_x, f_s, and μ_s^{min}



Find $a_x = ?$

① $\sum F_x = ma_x$

$$\sum F_x = mg\sin\theta - f_s = ma_x$$

$a_T = R\alpha$

② $\sum \tau = I\alpha$ (about the point O)

$a_T = a_x$

$$f_s R = \left(\frac{2}{3}MR^2\right)\left(a_T/R\right) \quad f_s = \frac{2}{3}Ma_T \Rightarrow f_s = \frac{2}{3}Ma_x$$

Substitute ② → ①

$$\sum F_x = \cancel{M}g\sin\theta - \frac{2}{3}\cancel{M}a_x = \cancel{M}a_x$$

$$g\sin\theta = \frac{5}{3}a_x \qquad \boxed{a_x = \frac{3}{5}g\sin\theta}$$

$3.62\ m/s^2$

Find $f_s = ?$

from ②, $f_s = \frac{2}{3}Ma_x = \frac{2}{3}M\left(\frac{3}{5}g\sin\theta\right)$ $\boxed{f_s = \frac{2}{5}Mg\sin\theta}$

Find $\mu_s^{min} = ?$

$$f_s^{max} = \mu_s^{min} N \qquad \mu_s^{min} = \frac{f_s^{max}}{N} = \frac{\frac{2}{5}Mg\sin\theta}{Mg\cos\theta} = \frac{2}{5}\tan\theta$$

$$\boxed{\mu_s^{min} = \frac{2}{5}\tan\theta}$$

b.) How would your answers to part (a) change if the mass were doubled?

$a_x = \frac{3}{5}g\sin\theta$ (Does not change)

$f_s = \frac{2}{5}Mg\sin\theta$ (f_s doubles)

$\mu_s^{min} = \frac{2}{5}\tan\theta$ (Does not change)

Chapter 10 In-Class Solutions

Ex. 32 An engine delivers 175 hp to the propeller at 2400 rpm.

a.) Torque = ? Power = Torque (ang. velocity) = $\tau \cdot \omega$

$$\tau = \frac{\text{Power}}{\omega} = \frac{175\text{ hp}\left(746\text{ W}/1\text{ hp}\right)}{2400\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = \boxed{519.4\text{ N}\cdot\text{m}}$$

b.) How much work does the engine perform in one revolution?

$$W = \tau\,\Delta\theta = (519.4\text{ N}\cdot\text{m})(2\pi\text{ rad}) = 3264\text{ N}\cdot\text{m}$$

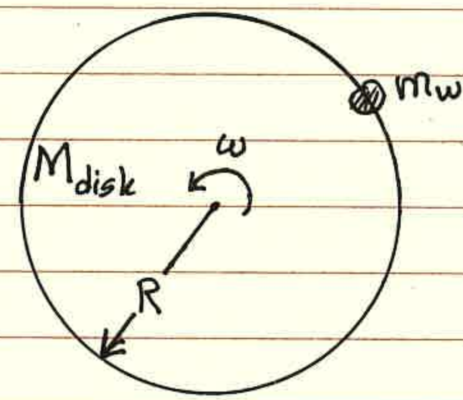
$$\boxed{W = 3264\text{ J}}$$

Ex. 38 A woman with a mass 50 kg is standing on the rim of a large disk that is rotating at 0.800 rev/sec . . . .

$m_w = 50$ kg

$M_{Disk} = 110$ kg

$R = 4.0$ m



$$L_{TOTAL} = I_{Disk}\,\omega + I_w\,\omega$$
$$= \left(\tfrac{1}{2}M_{Disk}R^2\right)\omega + m_w R^2\omega$$

$$L_{TOTAL} = R^2\omega\left(\tfrac{1}{2}M_{Disk} + m_w\right)$$
$$= (4.0\text{m})^2\left(0.800\,\frac{\text{rev}}{\text{s}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\right)(55\text{ kg} + 50\text{ kg}) = 8{,}445\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\boxed{L_{TOTAL} = 8{,}445\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

Chapter 10 In-Class Solutions

Ex. 43 Under some circumstances a star can collapse into an extremely dense object made mostly of neutrons, ...

Assume a uniform, solid, rigid sphere. $I_{cm} = \frac{2}{5}MR^2$

$R_i = 7.0 \times 10^5$ km $\quad T_i = 30$ days

$R_f = 16$ km $\quad \omega_f = ?$

Conservation of Angular Momentum $\Longrightarrow$ $L_i = L_f \quad I_i\omega_i = I_f \omega_f$

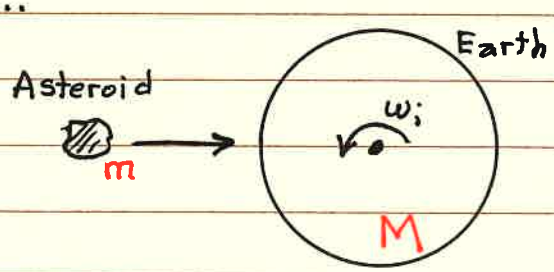
$$\omega_f = \omega_i \frac{I_i}{I_f} = \frac{2\pi}{T_i} \frac{\frac{2}{5}MR_i^2}{\frac{2}{5}MR_f^2} = \frac{2\pi}{T_i}\left(\frac{R_i}{R_f}\right)^2 = \frac{2\pi}{30\text{ days}\left(\frac{86{,}400\text{ s}}{\text{day}}\right)}\left(\frac{7.0\times 10^5}{16}\right)^2$$

$$\boxed{\omega_f = 4{,}640 \ \frac{\text{rad}}{\text{s}}} \times \frac{1\text{ rev}}{2\pi \text{ rad}} = 738 \ \frac{\text{rev}}{\text{sec}}$$

Ex. 48 Asteroid Collision! Suppose an asteroid is traveling straight toward the center of the earth, ...

$T_i = 1$ day $\qquad I_i = \frac{2}{5}MR^2$

$T_f = 1.25$ day $\qquad I_f = \frac{2}{5}MR^2 + mR^2$



Conservation of Angular Momentum $\Rightarrow L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f$

$$\frac{I_f}{I_i} = \frac{\omega_i}{\omega_f} \Rightarrow \frac{\frac{2}{5}MR^2 + mR^2}{\frac{2}{5}MR^2} = \frac{2\pi/T_i}{2\pi/T_f} = \frac{T_f}{T_i} = 1.25$$

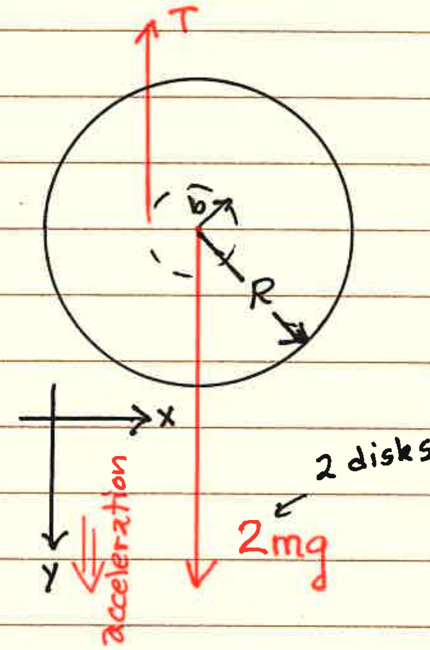
$$1 + \frac{5}{2}\frac{m}{M} = 1.25 \qquad \frac{5}{2}\frac{m}{M} = 0.25 \qquad \frac{m}{M} = \frac{2}{5}(0.25)$$

$$m = \frac{2}{5}(0.25)M = 0.1\ M \qquad \boxed{m = 0.100\ M}$$

DATE	
TOPIC	5

Chapter 10 In-Class Solutions

Prob. 71 The Yo-yo. A yo-yo is made from two uniform disks each with a mass m and radius R connected by a light axle of radius b.



$$I_{cm}^{disk} = \frac{1}{2} mR^2$$

① $\sum F_y = 2ma$

② $\sum \tau = I\alpha$

① $\sum F_y = 2mg - T = 2ma_y$

$$\boxed{T = 2m(g-a_y)}$$

② $\sum \tau = Tb = 2I_{cm}\alpha$ $\quad \alpha = \frac{a_T}{b}$

$$T = \frac{2\left(\frac{1}{2}mR^2\right)a}{b^2}$$

$$\boxed{T = \frac{mR^2 a_y}{b^2}}$$

$$2m(g-a_y) = \frac{mR^2 a_y}{b^2} \Rightarrow 2\cancel{m}g = \cancel{m}a_y\left(2 + \frac{R^2}{b^2}\right)$$

$$a_y = \frac{2g}{\left(2 + R^2/b^2\right)}$$

$a \to 0$, when $b \to 0$

① $T = 2m\left(g - \frac{2g}{2+R^2/b^2}\right) = 2mg\left(1 - \frac{1}{1+R^2/2b^2}\right)$

or

$$T = 2m\left(\frac{\cancel{2g} + R^2g/b^2 - \cancel{2g}}{2+R^2/b^2}\right) = \frac{2m\,R^2g/b^2}{2+R^2/b^2} \quad \div \frac{R^2/b^2}{R^2/b^2}$$

$$\boxed{T = \frac{2mg}{1 + 2\left(b/R\right)^2}}$$

Chapter 10 In-Class Solutions

Ex. 56 A certain gyroscope precesses at a rate of 0.50 rad/s when used on earth. $g_{moon} = 0.165\,g$ $\quad g = g_{earth} = 9.8\,\frac{m}{s^2}$

$$\Omega_E = \frac{mgr}{I\omega} \qquad \frac{\Omega_{moon}}{\Omega_E} = \frac{\frac{m g_{moon} r}{I\omega}}{\frac{mgr}{I\omega}} = \frac{g_{moon}}{g}$$

$$\Omega_{moon} = \frac{m g_{moon} r}{I\omega} \qquad \Omega_{moon} = \Omega_E \frac{g_{moon}}{g} = 0.50\,\frac{rad}{s}\left(\frac{0.165g}{g}\right)$$

$$\Omega_{moon} = 0.083\ rad/s \rightarrow \boxed{0.0825\ rad/s}$$ 3 sig. digits

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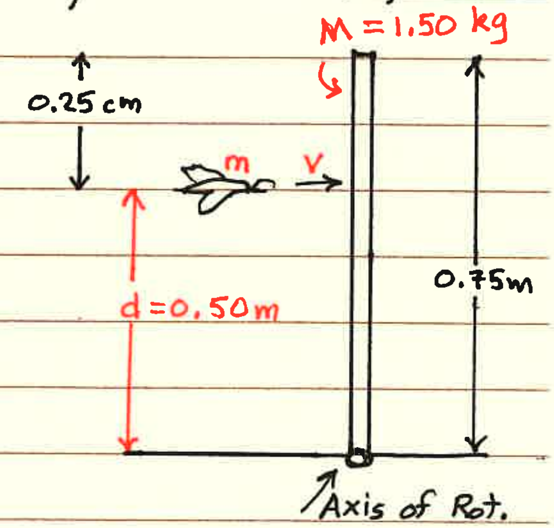
Prob. 85 A 0.500-kg (m) bird is flying horizontally at 2.25 m/s (v), not paying much attention, . . .



Conservation of Angular Momentum

$$L_i = L_f \qquad mvd = I\omega \qquad d = \tfrac{2}{3}L$$

$$\omega = \frac{mvd}{I} = \frac{mvd}{\frac{1}{3}ML^2} = \frac{3mvd}{ML^2}$$

$$\omega = \frac{2mv}{ML} = \frac{2(0.500\,kg)(2.25\,m/s)}{1.50\,kg\,(0.75\,m)}$$

$$\boxed{\omega_i = 2.00\ rad/s}$$

b.) $\omega_f = ?$ Cons. of Energy $\Rightarrow$ Work-Energy Theorem

$$W_{gr} = \Delta K = K_f - K_i = \tfrac{1}{2}I(\omega_f^2 - \omega_i^2) \Rightarrow Mg\left(\tfrac{L}{2}\right) = \tfrac{1}{2}I(\omega_f^2 - \omega_i^2)$$

$$Mg\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)(\omega_f^2 - \omega_i^2) \Rightarrow (\omega_f^2 - \omega_i^2) = \frac{MgL/2}{\frac{1}{6}ML^2} = \frac{3g}{L}$$

$$\omega_f^2 = \omega_i^2 + \frac{3g}{L} = \left(2.0\,\frac{rad}{s}\right)^2 + \frac{3(9.8)}{0.75} = 43.2\,\frac{rad^2}{s^2} \qquad \boxed{\omega = 6.57\,\frac{rad}{s}}$$