The Zeeman Effect

Many elements of this lab are taken from "Experiments in Modern Physics" by A. Melissinos.

Intro

- **The mercury lamp may emit ultraviolet radiation that is damaging to the cornea: Do NOT look directly at the lamp while it is lit. Always view the lamp through a piece of ordinary glass or through the optics chain of the experiment. Ordinary glass absorbs ultraviolet radiation. The mercury lamp's housing however may be quartz glass which doesn't absorb the ultraviolet.**
- Don't force anything.
- If you are unsure, ask first.

In this lab, you will measure the Bohr Magneton by observing the Zeeman splitting of the mercury emission line at 546.1 nm. To observe this tiny effect, you will apply a high-resolution spectroscopy technique utilizing a Fabri-Perot cavity as the dispersive element. The most difficult aspect of this experiment is the interpretation of the Fabri-Perot cavity output and understanding the underlying theory.

It's easy to confuse the many different "effects" modifying the energy of electrons orbiting the nucleous (and therefore affecting atomic spectra) so here is a short guide to the various types: [1]

- *Basic level structure* – This is the familiar structure most of us learn first in chemistry class whereby electrons arrange themselves in orbital "shells" around the nucleus. The orbitals have a number assigned to them, corresponding very roughly to their distance from the nucleus: 1, 2, 3 etc. They also have a letter assigned to them corresponding to the shape of the orbital: S, P, D, .. etc (S shells are spherical, P shells look like two teardrops with the pointy ends touching, etc.) Each S shell can hold 2 electrons, each P shell can hold 6, each D shell 10 and so on. The energy of each level depends on whether or not all lower levels are filled and/or on how many electrons are present in each level. When all lower levels are occupied, the order of the levels in terms of energy is: 1S, 2S, 2P, 3S, 3P, 4S, 3D, 4P, etc. The basic structure we see in spectroscopy is due to transitions of the outermost electrons to, from, or between higher unoccupied shells of the atom. The number of the shell is known as the principle quantum number n=1,2,3,... The letters S,P,D,... on the other hand correpsond to different (and quantized) orbital angular momenta labelled by the quantum number l=1,2,3,... respectively. Usually, exposure to these quantum numbers first occurs when studying the Hydrogen atom in a Quantum Mechanics course.

- *Fine Structure* – Since the electrons travel quite fast while "orbiting" the nuclei, relativistic effects can be important. Relativistic effects cause small shifts in the energy levels (relativistic corrections to the kinetic and potential energies) and also lead to splitting of levels due to a relativistic interaction between the orbital angular momentum of the electron and its intrinsic angular momentum (or spin). Electrons in the same orbital but in opposite spin states will have slightly different energy levels.

[1] Extracted from "Physics of Atoms and Molecules" by B.H. Bransden and C.J. Joachain.

This last effect is known as spin-orbit coupling.

- *Lamb shift* – Slight shifts in the energy levels due to vacuum fluctuations that "smear out" the electron, making it sensitive to the non-uniformity of the fields surrounding he nucleous.

- *Hyperfine structure* – This is due to coupling of the electron energy levels with the intrinsic angular momentum (spin) of the nucleous. States of the electron with differing angular momenta will be further split by different levels of coupling with the nuclear spin.

- *Volume effect* – Slight shifts in the expected energy levels due to the fact that the nucleous has finite size.

- *Zeemann effect* – The splitting of energy levels with different angular momenta when an external magnetic field is applied. Since an electron with non-zero angular momentum must have a non-zero magnetic dipole moment, this dipole moment will interact with any environmental magnetic dipole field, thus changing the energy of the electron by an amount that depends on its angular momentum.

- *Stark Effect* – The splitting of energy levels due to the application of an external electric field.

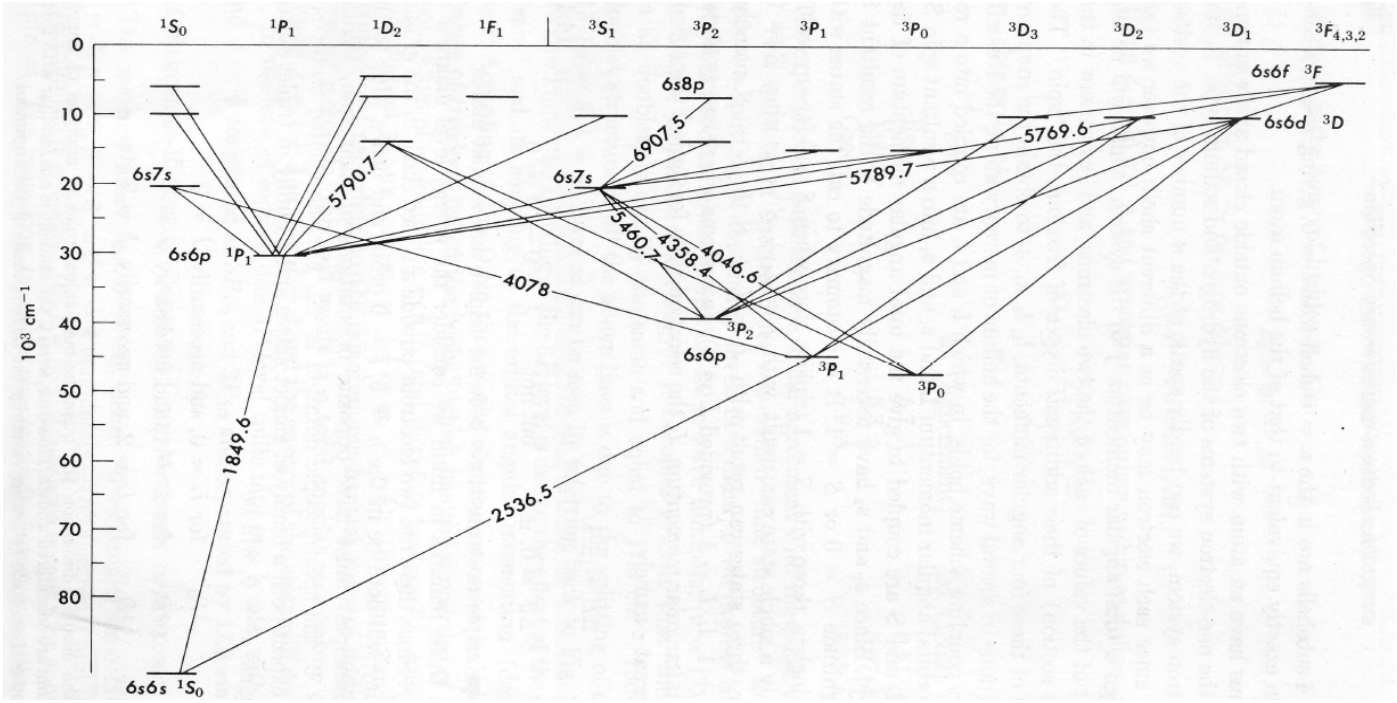
The Mercury atom, Hg, has 80 electrons. The first four (n=1,2,3,4) levels are full holding at total of 60 electrons, the n=5 level has the S,P and D, shells occupied (l=0,1,2) holding another 18 electrons. The two final electrons occupy the 6S shell (n=6, l=0). We can think of the 58 inner electrons as being "inert" in that they act as a closed system that doesn't interact with the outermost 2 electrons (except to passively sheild the nucleus, modifying the nuclear potential slightly so that the outermost electrons don't quite see a $1/r$ potential). When Hg_{198} gas is excited to discharge by an RF field as in our experiment, numerous transitions of the outer two electrons are observed. We will use a narrow pass-band filter known as an interference filter to isolate the green line at 546.07 nm. This spectral line corresponds to a transition of one of the electrons between the 7S shell and the 6P shell, while the other electron remains in the 6S shell. What we will see is that in the presence of a magnetic field, the line due to this transition will split into several different lines. This is an example of the Zeeman effect. We now consider the slitting of this transition in a quantitative way.

Because of the closed nature of the inner shells, Mercury is a fairly simple multi-electron atom behaving, from a spectroscopic standpoint, rather like Helium. The two electrons in the outermost shell don't act independently but are coupled (according to the so-called Russell-Saunders coupling scheme) so that the *combined* orbital angular momentum **L** of the electrons has a well defined value, as does the *combined* spin **S**. This is a consequence of the fact that **L** and **S** for the *coupled* electron pair both commute with the Hamiltonian **H** even though the angular momenta for the individual electrons does not. Therefore whenever the *coupled* electron system has a well defined energy, the coupled system will also have a well defined total orbital angular momentum and a well defined total spin even though we can't know the orbital angular momenta or spins of the electrons *individually*. The eigenstates of **L** and **S** are labelled by the quantum numbers l and s respectively.

You may be wondering whether the orbital angular momenta and the intrinsic angular momenta of the coupled electron state don't just add up to one overall total angular momentum. Indeed they do. The total angular momentum of the coupled electrons is $\mathbf{J} = \mathbf{L} + \mathbf{S}$ which commutes with the Hamiltonian just like **L** and **S**. So, since the combined electron system has well defined

values of orbital and intrinsic angular momentum in each of the two states involved in the transition, the value of the total angular momentum **J**, is also well defined in each state. The eigenstates of **J** (which are also eigenstates of **L** and **S**) are labelled by the quantum number *j*, *which can take on different values depending on whether the total spin of the two coupled electrons is aligned or anti-aligned with the orbital angular momentum:* $j = l \pm s$. Note that *l* and *s* correspond to the magnitude of the orbital momentum and spin and are themselves therefore always positive.

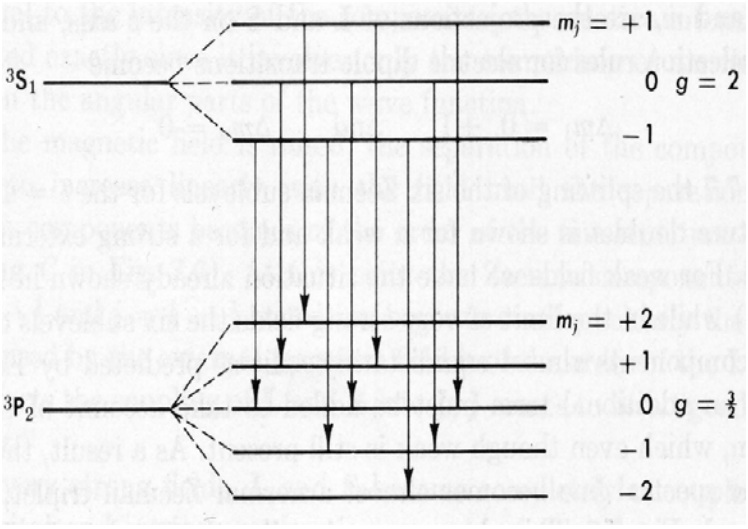
The upshot of this whole discussion is that each state of the coupled system consisting of the two outermost electrons of mercury corresponds to a unique combination of the quantum numbers *l, s* and *j* for the coupled system. Therefore, transitions leading to spectral lines can be identified as being transitions between the states of particular combinations of the quantum numbers *l, s* and *j*. The transitions giving some of the strongest spectral lines in mercury are shown on the level diagram below (from Melissinos, 1st ed). The various states are indicated by short horizontal lines whos position on the y-axis indicates the energy of the state. The states are arranged in columns according to their quantum numbers, *l, s* and *j* where *l, s* and *j* are indicated using the traditional spectroscopic notation. The superscript on the left of the captial letter is 2*s*+1. The capital letter indicates the value of *l*: The letter S corresponds to *l*=0, P corresponds to *l*=1, D to *l*=2, F to *l*=4, etc. The subscript is the quantum number *j*. [In the diagram below, the lines themselves are also marked by the shells occupied by the electrons. Although this information is strictly speaking superfluous it helps to identify what is going on.] The diagonal lines between the states indicate allowed transitions and numbers indicate the wavelength of the transition in Angstroms.



The 546.07 nm transition we are investigating is between the states $^3S_1$ and $^3P_2$. Therefore, (translating the spectroscopic notation) the initial combined *l* for the two coupled outer electrons is 0 while the initial combined spin *s* is 1 (+1/2 from each electron). The final *l* (after one

electron has jumped from the 7S to the 6P shell) is 1 while the combined spin is still 1 where the combined spin points in the same direction as the orbital angular momentum.[2]

It is an interesting feature of quantum mechanics that an eigenstate of the total angular momentum **J** is not simultaneously an eigenstate of all the *vector components* of the angular momentum. In other words, we can know the length of the angular momentum vector but not its direction. In transition being discussed here, for example, there are several possible values of the z-component of angular momentum $\mathbf{J_z}$ for the both the initial state and final states. The eigenstates of $\mathbf{J_z}$ are labelled by the quantum number m_j, which can take values $m_j = -j,-j+1,..j-1,j$. So, for the initial state in our transition (6S,7S: $l=0$, $s=1 \Rightarrow j=l+s=1$) there are three possible values of m_j: -1, 0, +1. Similarly for the final state (6S,6P: $l=1$, $s=1 \Rightarrow j=l+s=2$) there are five possible values of m_j: -2,-1,0,1,2. In the absence of a magnetic field, states differing only in the quantum number m_j all have the same energy (are degenerate). However, in the presence of a magnetic field the degeneracy is broken. Taking the z-axis to be along the axis of the magnet in our experiment , you can see that states with differing values of m_j will have a different z-component of the magnetic moment. Since it is this magnetic moment that interacts with any external field, the energies of states with different m_j's being otherwise identical will differ when a magnetic field is present. From this argument, we might expect that when we turn on the field or single line at 546.07 nm would split into 3x5=15 lines corresponding to transitions between states of each different m_j. However, this is not the case. Instead, we see 9 lines. This is because transitions with $|\Delta m_j| > 1$ are not present. This is an example of a so-called "selection rule". The selection rule $|\Delta m_j| > 1$ is related to the fact that the photons that must be emitted to allow the transition to occur (hence the spectral lines!) must be emitted as electric dipole radiation. The allowed transitions are shown in the figure below



The splitting of the lines is given by the strength of the external field and by the intrinsic magnetic moment of the coupled electron state. The magnetic contribution to the total energy is given by

$$\Delta E = g\mu_0 B m_j$$

[2] As an aside, note that Pauli exclusion doesn't require opposite electron spins since the two electrons are never both in the same state, e.g. 7S goes to 6P, but not to 6S.

where μ_0 is the Bohr Magneton

$$\mu_0 = \frac{e\hbar}{2m_e}$$

and g is the Lande g-factor

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}.$$

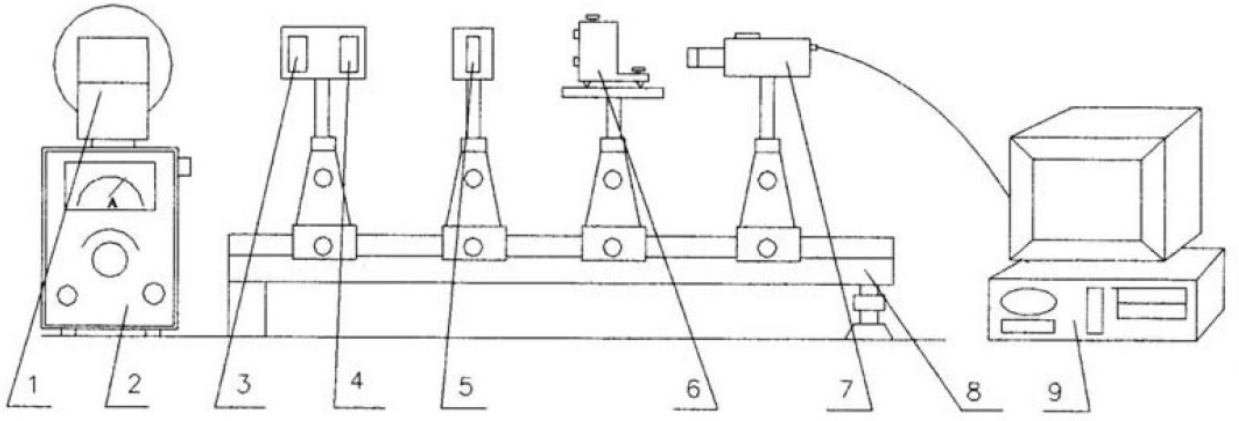
By measuring the frequency shift between the adjacent rings of the split 546.07 nm line of mercury in a magnetic field, we obtain an estimate for ΔE. You will know B independently and you can calculate the initial and final g's and m_j's from the considerations above. Thence, you should obtain a value for the Bohr Magneton, μ_0.

Tasks

Concentrating on the three central lines in the multiplet (corresponding to Δm_j=0), make a plot of the frequency shift of each of these lines from the frequency of the original no-field line. versus magnetic field strength. By fitting these frequency shifts to the form expected from the equations above, measure the value (and uncertainty) of the Bohr magneton.

Hints:

Setup the apparatus as shown in the diagram and described in the manufacturer's hardware setup documentation. Use the camera to record photographs as a function of field strength. Make sure you note the magnet current for each photograph. Obtain a calibration for the electromagnet from the instructor to allow you to convert this to magnetic field strength. **Note: Do not leave the magnet current above 0.7 Amps for more than about 10-15 seconds. The magnet overheats easily and could be ruined.** You will use these photographs to analyze the Zeeman splitting as a function of field strength. Note that the polarization varies between different transitions in the multiplet allowing you to eliminate all but the subset of the transitions that you intend to study.



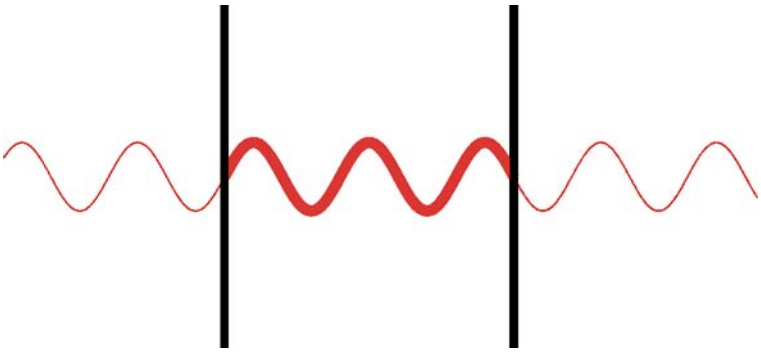
1. Electromagnet (Max. 1.3 T)
2. Control unit
3. Lens (f=15cm)
4. Polarizer
5. Interference filter (peak transmission wavelength 546 nm)
6. Fabri-Perot cavity (flat mirrors)
7. CCD camera
8. Optical rail
9. Computer to display and save CCD camera images.

Understanding the Operation of the Fabri-Perot Cavity

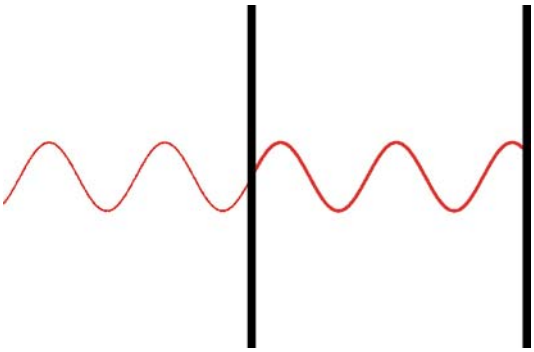
A short summary of the operation of the Fabri-Perot is given below. However, you will find that A. Melissinos, "Experiments in Modern Physics" has more detail. Together, these two resources should enable you to interprete your data. Copies of Melissinos are available in the laboratory. Be aware that in an effort to make things clearer I've chosen to use slightly different notation than Melissions! In particular, note that I have chosen to define n_0 as twice the number of wavelengths required to span the Fabri-Perot cavity, rounding down to the nearest integer when the length of the cavity is not an integer number of wavelengths. In other words, in my notation, n_0 is the number of *whole* wavelengths required for a round trip passage of the cavity at normal incidence. Melissinos however defines n_0 by $n_0 \equiv 2t/\lambda$ where *t* is the length of the cavity. This is generally not an integer. In Melissinos therefore, the quantity $n_0 - \varepsilon$ is the integer corresponding to my n_0. Also, I chose to call the innermost transmitted ring, the zero'th ring while Melissinos calls it the first ring. Therefore, Mellisinos' definition of the ring number *p* differs from mine by one. Despite these differences, our definitions of the important quantity ε (which enters into your data analysis) are the same. So, in the end it doesn't matter whether you chose to follow my exposition below or that of Melissinos.

The Fabri-Perot cavity consists of two parallel semi-transmissive mirrors placed a fixed distance apart. The cavity thus formed is non-transmissive to light that is perpendicularly incident on the cavity *unless* the distance between the plates is an integer number of half wavelengths. When the distance between the plates is an integer number of half wavelengths, any number of round-trip (back and forth) traversals of the cavity brings the light it back into phase with the incident wave. The incident wave and all subsequent round-trip waves in the cavity will therefore add constructively to form a standing wave in the cavity. Because it is built up with multiple reflections adding coherently, the standing wave inside the cavity is much more intense than the entering wave. Although the right hand mirror may only be slightly transmissive, the intensity of the standing wave inside the cavity is sufficiently high that the intensity of light leaking through the right hand mirror, and constituting the transmitted beam, is fairly significant. In fact, one can show that the intensity of the transmitted beam is close to that of beam incident on the cavity.

The diagram below shows a transmissive Fabri-Perot cavity with 5 half-wavelengths fitting in the cavity forming a standing wave. In the diagram, light is incident from the left.



The diagram below shows a non-transmissive Fabri-Perot cavity with a non-integer number of half-wavelengths in the cavity and thus no standing wave. Note that the right hand mirror is shifted left compared with the above diagram, so the cavity is slightly shorter than 5 half-wavelengths.

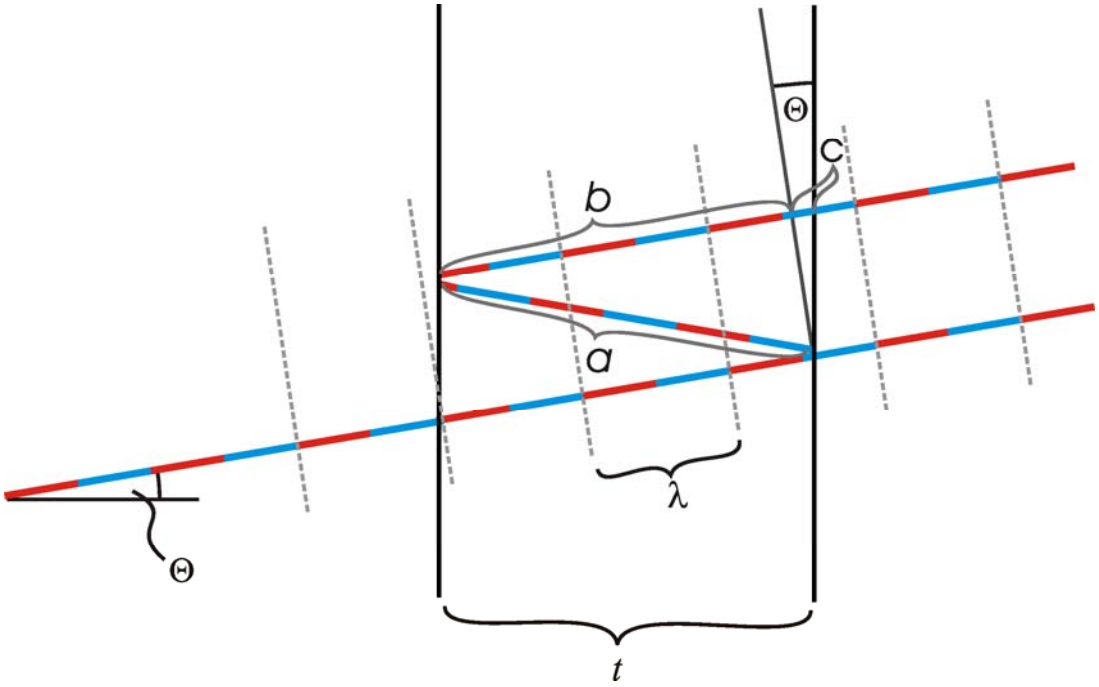


However, light rays incident on the cavity are not always perpendicular to the cavity mirrors. If the source of light is not perfectly collimated, some light rays may be incident on the cavity at an angle. For example, a point-like light source will emit a spherical phasefront. These light rays can also be transmitted through the cavity provided that a round-trip in the cavity brings them back into phase with the original beam.

This situation is illustrated in the diagram below where the original wave is incident from the red and blue (light and dark) intervals represent half-wavelengths. The dotted lines represent phasefronts (planes of equal phase) while θ is the angle of incidence. The cavity transmits when the angle θ is such that the distance $a+b$ is equal to an integer number of wavelengths

$$a + b = n\lambda$$

In the diagram below n=5. When the angle θ is such that this condition is satisfied, all the beams reflecting back and forth within the cavity are in phase as they travel to the right. Therefore they interfere constructively forming a standing wave in the cavity. As in the case of normal incidence, the standing wave inside the cavity is much more intense than the beam entering the cavity. This causes the transmitted beam to have an intensity close to that of the incident beam. In contrast, rays entering at angles close to, but not equal to, θ will not experience constructive interference, no standing wave will be formed, and the cavity will transmit those rays extremely weakly or not at all.



It may seem unlikely to you that a standing wave could be produced by beams travelling at an angle like this to the cavity faces. After all, the forward traveling rays inside the cavity above don't actually appear to overlap. So how can a standing wave be formed? Remember that this is all supposed to be happening at small angles and the diagram above greatly exagerates the angles. In reality all the angles in question are much smaller, so subsequent reflected waves within the cavity do in fact *mostly* overlap and do therefore form a standing wave (provided of course that, as discussed, a traversal of the cavity serves to bring the wave back into phase with the incoming wave). Similarly, the number of waves *n* corresponding to the distance *a*+*b* is much greater in reality than the 5 waves shownin the diagram. For our 2cm long cavity, a realistic *n* is an integer around $4\text{x}10^4$.

The left hand side of the above criterion above can be rewritten

$$a+b = t/\cos(\theta)+\left[t/\cos(\theta)\right]\cos(2\theta) = t\frac{1+\left[2\cos^2(\theta)-1\right]}{\cos(\theta)} = 2t\cos(\theta)$$

leading to

$$2t\cos(\theta_n) = n\lambda$$

where we have replaced θ with θ_n to emphasize that there are multiple angles satisfying this equation, one for each of the integers n. Therefore, if the incoming beam contains rays at all angles, such as rays exiting from a point-like source, many transmission will be evident at a number of angles. A beam exiting from a point-like source will have spherical wavefronts even after passing through any combination of spherical lenses. Such a beam consists of rays whose angle of incidence is proportional to the distance from the optical axis. The transmission pattern of the cavity will therefore consist of a number of concentric circles whose spacing is depends on the radius of curvature of the incoming phasefronts.

Note that since $0 < \cos(\theta_n) < 1$, the number of wavelengths *n* goes *down* as we go from the inner rings to the outer rings. If a normally incident ray completes a total number n_0 of *whole*

wavelengths during a round trip in the cavity, then for the p^{th} transmitted ring (counting outwards from the center and calling the innermost ring the 0^{th} ring)

$$n = n_p \equiv n_0 - p$$

n_p is known as the "order" of the p^{th} ring. However, it is unlikely that the spacing of the plates is a precise integer multiple of the wavelengh. Therefore, a normally incident ray is not likely to be transmitted.[3] The total round trip length $2t$ of the Fabri-Perot cavity can be written as

$$2t = (n_0 + \varepsilon)\lambda$$

where n_0 is the round-trip length of the cavity in wavelengths (for normal incidence) rounded down to the nearest integer, and ε is the leftover fraction of a wavelength. The quantity ε is sometimes known as the "fractional order" and is an important quantity for the purposes of this lab.

Since we are in the small angle approximation, we can rewrite the criterion for transmission

$$\begin{gathered} 2t\cos(\theta_n) = n\lambda \\ \Downarrow \\ 2t\cos(\theta_p) = (n_0 - p)\lambda \\ \Downarrow \\ 1 - \frac{{\theta_p}^2}{2} = \frac{(n_0 - p)\lambda}{2t} \end{gathered}$$

where we have also re-labeled the transmission angle according to the number of the corresponding ring p. We can rewrite $n_0 - p$

$$n_0 - p = (n_0 + \varepsilon) - p - \varepsilon = \frac{2t}{\lambda} - p - \varepsilon$$

And substituting back into the previous expression and solving for θ_p gives us the angle of transmission of the p^{th} ring

$$\theta_p = \frac{\lambda}{2t}\sqrt{p + \varepsilon}$$

If the radius of curvature of the beam incident on the Fabri-Perot cavity is R, and we were to measure the radius of subsequent concentric circles at the output plate of the cavity (don't try this!) their radii would be given by

$$r_p = \mathcal{R}\tan(\theta_p) \approx \theta_p = \frac{\mathcal{R}\lambda}{2t}\sqrt{p + \varepsilon}$$

I've assumed that the length t of the cavity is much smaller than R and that the small angle approximation holds.

---

[3] However, there will of course still be angles θ_n which do form standing waves, so the ring pattern does not depend on the cavity being an integer number of wavelengths long.

We're almost there. Recall that the action of a spherical lens or any combination of spherical lenses is ultimately simple. It is to turn an incoming ray through an angle proportional to the distance from the lens axis. Therefore, when we use a lens to image the output of the Fabri-Perot cavity onto a CCD, all we are ultimately doing is multiplying the right hand side of the above equation by some constant factor *C'*. We might as well absorb the entire factor out front into a constant *C*. In other words, the position of the concentric rings on the CCD or TV screen is just

$$R_p = C\sqrt{p+\varepsilon}$$

By fitting R_p versus p to the above equation we can extract both C and the fractional order ε. Finally, recall that

$$\varepsilon = \frac{2t}{\lambda} - n_0$$

So, if two very close wavelengths λ_1 and λ_2 enter a Fabri-Perot cavity, so close that they both have the same n_0, then we can calculate their difference using

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{2t}\left(\varepsilon_1 - \varepsilon_2\right).$$