

Prob. 86

## Neutron Star and Supernova Remnants

The Crab Nebula is a cloud of glowing gas  $\sim 10$  light years across.  
(1054 AD)

$$P = \text{power} = 5 \times 10^{31} \text{ W}$$

$$K_{ns} = \frac{1}{2} I_{ns} \omega^2$$

$$\omega = \frac{2\pi}{T}$$

$$I_{ns} = \frac{2}{5} MR^2$$

Given:  $T = 0.0331 \text{ s}$   $\frac{dT}{dt} = 4.22 \times 10^{-13} \frac{\text{sec.}}{\text{sec}}$

$$K_{ns} = \frac{1}{2} I_{ns} \left( \frac{2\pi}{T} \right)^2 = \frac{4\pi^2}{2} I_{ns} \frac{1}{T^2}$$

from chain-rule

$$\text{Power} = \frac{dK_{ns}}{dt} = 2\pi^2 I_{ns} \frac{d(T^{-2})}{dt} = 2\pi^2 I_{ns} (-2T^{-3}) \left( \frac{dT}{dt} \right)$$

$$\text{Power} = -4\pi^2 \frac{I_{ns}}{T^3} \left( \frac{dT}{dt} \right) \quad I_{ns} = \frac{\text{Power } T^3}{4\pi^2 \frac{dT}{dt}} = \frac{(5 \times 10^{31} \text{ W})(0.0331 \text{ s})^3}{4\pi^2 (4.22 \times 10^{-13} \text{ s/s})}$$

$$I_{ns} = 1.088 \times 10^{38} \text{ kg} \cdot \text{m}^2$$

b.)  $M_{ns} = 1.4 \text{ solar masses} = 1.4 (1.99 \times 10^{30} \text{ kg})$

$$I_{ns} = \frac{2}{5} M_{ns} R^2 \quad R^2 = \frac{5 I_{ns}}{2 M_{ns}} \quad R = \sqrt{\frac{5}{2} \frac{1.088 \times 10^{38} \text{ kg} \cdot \text{m}^2}{1.4 (1.99 \times 10^{30} \text{ kg})}}$$

$$R = 9881 \text{ meters} \quad \sim 10 \text{ km}$$

c.)  $v_T = R\omega = R \frac{2\pi}{T} = 9881 \text{ m} \left( \frac{2\pi}{0.0331 \text{ s}} \right) \quad v_T = 1.88 \times 10^6 \text{ m/s}$

$$\frac{v_T}{c} = \frac{1.88 \times 10^6 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} = 6.25 \times 10^{-3} \quad 0.625\% \text{ of } c$$

speed of light  $\rightarrow c$

d.)  $\text{Density} = \frac{M_{ns}}{\text{Volume}} = \frac{1.4 (1.99 \times 10^{30} \text{ kg})}{\frac{4}{3} \pi (9881 \text{ m})^3} \quad \text{Density} = 6.89 \times 10^{17} \text{ kg/m}^3$