Chapter 4
Newton's Laws of Motion

Up until now, we have been investigating the field of physics called *kinematics*, the physical properties of *space* and *time*. In this chapter, we will introduce the concept of inertia (or mass) and begin our journey into the subject of *dynamics*, the relationship of motion due to the forces that cause it.

1 Force and Interactions

There are two kinds of forces, *contact* forces, and *action-at-a-distance* forces, sometimes called long-range forces. Contact forces are easily identifiable because, as the name implies, there is an obvious *push/pull* force making contact with the system of interest. In the case of *long-range* forces, we will introduce the concept of a *field* to describe the force, along with its strength and direction.

Gravity is an example of an *action-at-a-distance* distance force.

Force is a **vector quantity**. It has both *magnitude* and *direction.* The SI units of force is the *newton* (N). Just as we did with position, velocity, and acceleration vectors, we will write force vectors in *component* form.

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

Example: Components of the force vector $\vec{F}$

1.1 Superposition of Forces

When two or more forces $\vec{F}_1$, $\vec{F}_2$, etc., act at the same time on the same point of a body, experiments show that the motion resulting from these multiple forces can be produced by a single force $\vec{R}$, the vector sum of the original forces:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \cdots$$

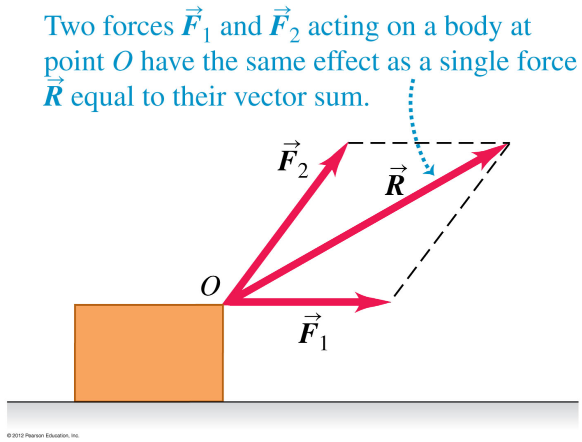


Figure 1: Superposition of Forces

This is called the principle of *superposition*. The net force $\vec{R}$ acting on a body is defined to be:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum \vec{F} \tag{1}$$

As before, we will want to take this vector equation and examine its scalar components $\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$.

$$R_x = \sum F_x \qquad\qquad R_y = \sum F_y$$

where the magnitude of $\vec{R}$ is:

$$R = \sqrt{R_x^2 + R_y^2} \qquad\qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \quad \text{(in 2 dimensions)}$$

Ex. 4: A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of 20.0^o, and the man pulls upward with a force $\vec{F}$ whose direction makes an angle of 30.0^o with the ramp (Fig. E4.4). a) How large a force $\vec{F}$ is necessary for the component F_x parallel to the ramp to be 90.0 N b) How large will the component F_y perpendicular to the ramp then be?

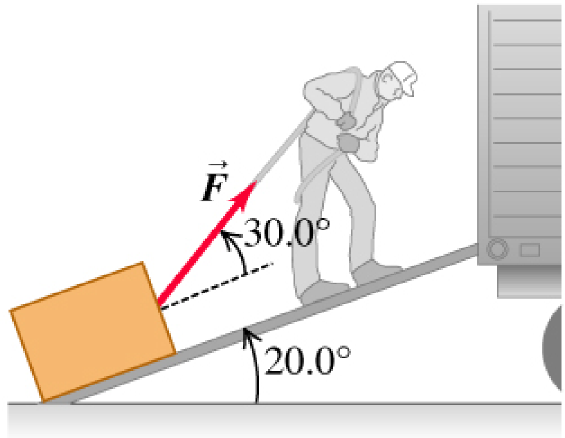


Figure 2: Figure E4.4 from University Physics).

2 Newton's First Law

Experiments have confirmed, time and again, the wide application of this physical law.

Newton's 1$^{\text{st}}$ Law
A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

The tendency of a body to keep moving once it is set in motion results from a property called inertia. Sometimes, Newton's 1$^{\text{st}}$ law is called the *law of inertia.*

Newton's 1$^{\text{st}}$ law can be written mathematically as:

$$\sum \vec{F} = \vec{0} \qquad \text{(body in translational equilibrium)}$$

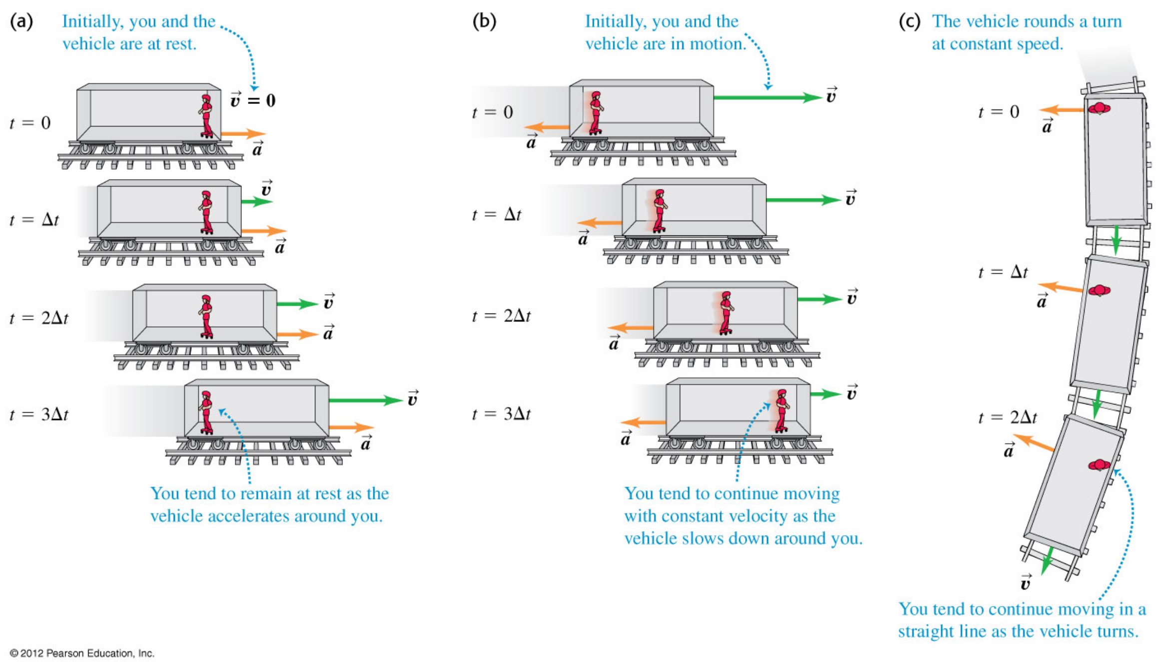
2.1 Inertial Frames of Reference

Newton's 1$^{\text{st}}$ law is valid only in inertial frames (i.e., constant-velocity frames). The earth is approximately an inertial frame.

The Galilean transformation of velocities:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

is only valid between inertial frames. In other words, the velocity of frame "B" with repsect to "A" ($\vec{v}_{B/A}$) must be constant.



3 Newton's Second Law

In this section we investigate the situation where $\sum \vec{F} \neq \vec{0}$. From experiments, we observe that when $\sum \vec{F} \neq \vec{0}$, that $\sum \vec{F}$ is proportional to the acceleration. In other words, the greater the "net force", the greater the acceleration. The physical quantity that links *force* to *acceleration* is called the mass.

$$\sum \vec{F} \propto \vec{a} \qquad\qquad \sum \vec{F} = m\vec{a}$$

where m is called the mass. The *mass* is the measure of inertia, or the *resistance* to acceleration.

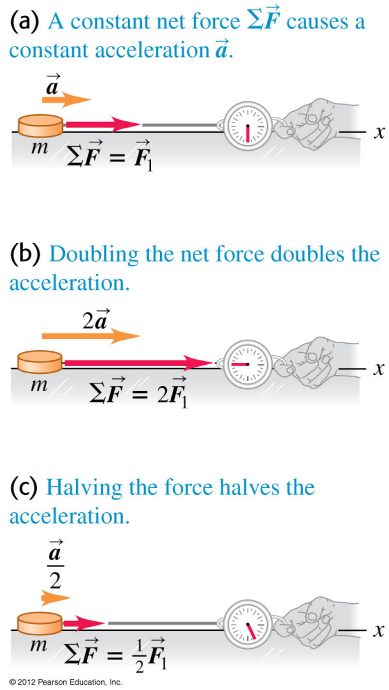


Figure 3: Acceleration: Doubling the force, doubles the acceleration.

Rewriting the above equation in scalar form, we can see how the *mass* is a measure of the resistance to acceleration:

$$m = \frac{\left|\sum \vec{F}\right|}{a}$$

If the acceleration resulting from a large force ($|\sum \vec{F}|$) is small, then obviously, the mass must be *large*.

Mass is a quantitative measure of inertia, the resistance to acceleration (or change in velocity). The SI units of mass is the *kilogram*. Until November 16, 2019, the kilogram was determined from a platinum-iridium right-cylinder kept in a vault near Paris, France. This object is still used to calibrate masses between laboratories because it is accurate to one-part in 10^8. However, the new kilogram is more precise and is determined from: (1) Planck's constant, $h = 6.626 \times 10^{-34}$ joules $\cdot$ sec, and (2) the meter and the second which are defined in terms of the speed of light (c) and a specific atomic transition frequency in *cesium*, $\Delta\nu_{\mathrm{Cs}}$. An alternative (and historical) definition of the kilogram was determined from Avogadro's number of isotopically pure (^{12}C) which should be equal to 12.00000 grams (exactly). However this had limited precision because there were fewer than 8 significant digits in Avogadro's number.

Now that we have defined the units of mass, we're ready to define the units of force. The force required to accelerate a one-kilogram mass 1.00 m/s^2 is 1.00 newtons (N).

$$1\,\mathrm{N} = 1\mathrm{kg}\cdot\mathrm{m/s}^2$$

Newton's 2$^{\text{nd}}$ Law
If a net external force acts on a body, the body accelerates. The direction of acceleration is the same as the direction of the net force. The net force vector is equal to the mass of the body times the acceleration vector of the body.

$$\sum \vec{F} = m\vec{a} \qquad \text{(Newton's 2}^{\text{nd}}\text{ Law)} \tag{2}$$

The acceleration resulting from the vector sum of forces can be written as:

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

Usually we don't use the vector form of Newton's 2$^{\text{nd}}$ law when applying it to physics problems. It's easier to use the scalar equations derived from Eq. 2 to determine the accelerations a_x, a_y, and a_z.

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

Up until now, we have either been *handed* the accelerations, or we determined the accelerations from other kinematical quantities. With Newton's 2$^{\text{nd}}$ law, we can now determine the accelerations, a_x, a_y, and a_z, from the forces acting on the body (i.e., the dynamics).

3.1 Some Notes on Units

In cgs units (centimeter, grams, seconds), the unit of force is called the *dyne*.

$$1\text{ dyne} = 1\text{ g}\cdot\text{cm/s}^2$$

In the British system of units, the unit of force is the *pound*.

$$1\text{ pound} = 1\text{ slug}\cdot\text{ft/s}^2$$

where a *slug* is a unit of mass in the British system. A 1-kilogram object weighs about 2.2 lb at the earth's surface.

Ex. 7: A 68.5 kg skater moving initially at 2.40 m/s on rough horizontal ice comes to rest uniformly in 3.52 s due to friction from the ice. What force does friction exert on the skater?

Ex. 12: A crate with mass 32.5 kg initially at rest on a warehouse floor is acted on by a net horizontal force of 14.0 N. (a) What acceleration is produced? (b) How far does the crate travel in 10.0 s? (c) What is its speed at the end of 10.0 s?

4 Mass and Weight

Objects near the earth's surface experience roughly the same acceleration due to the earth's gravitational "pull." Applying Newton's 2**nd** Law to objects near the earth's surface, we see that the *force due to gravity* is ma_y, or just mg. This force due to gravity is what we call the *weight*, W.

$W = mg$ (the weight of a body of mass m near the earth's surface)

where $g = 9.8$ m/s^2. For example, a 1 kg mass would weigh 9.8 newtons. Also recall that there are ~ 4.4 newtons to 1 pound.

Since the weight of an object is a force, it can also be written in vector form:

$$\vec{w} = m\vec{g}$$

Ex. 17: Superman throws a 2400-N boulder at an adversary. What horizontal force must Superman apply to the boulder to give it a horizontal acceleration of 12.0 m/s^2?

4.1 Variation of g with Location

AIP Handbook, 2nd edition Local values of g as a function of latitude.

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γ[ϕ_] = 978.0490 (1 + 0.0052884 Sin[ϕ Degree]^2 - 0.0000059 Sin[2 ϕ Degree]^2)
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$$978.049\left(1+0.0052884\,\mathrm{Sin}[°\,\phi]^2-5.9\times10^{-6}\,\mathrm{Sin}[2\,°\,\phi]^2\right)$$

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 980.359, 980.449, 980.539, 980.629, 980.72, 980.81, 980.9, 980.989, 981.079,
 981.167, 981.255, 981.343, 981.429, 981.515, 981.599, 981.682, 981.764,
 981.845, 981.924, 982.001, 982.077, 982.151, 982.224, 982.294, 982.362,
 982.429, 982.493, 982.554, 982.614, 982.671, 982.725, 982.777, 982.827,
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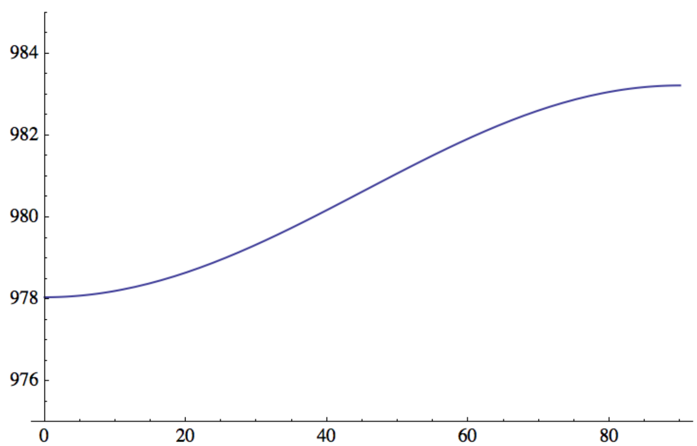


4.2 Measuring Mass and Weight

An equal-arm balance determines the mass of a body by comparing its weight to a known weight. Gravity must be present in order for the balance to function properly.

There are principally two methods used to determine the mass of an object:

the inertial mass: determined by applying an external force (other than gravity)

$$m_{\text{inertial}} = \frac{F}{a} \qquad \text{and}$$

the gravitational mass: determined by weighing the object and using the local value of g:

$$m_{\text{grav}} = \frac{w}{g}$$

5 Newton's Third Law

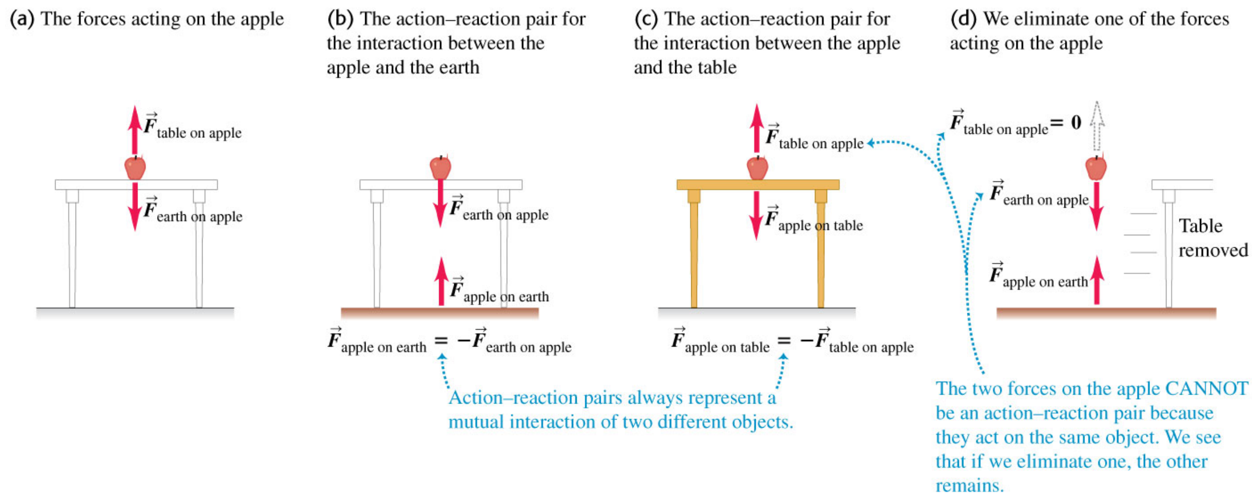
In Newton's first 2 laws, we found the acceleration resulting from the sum of external forces acting on a body. In the first case, $\vec{a} = \vec{0}$, and in the second case, $\vec{a} = \sum \vec{F}/m$. In Newton's 3$^{\text{rd}}$ law, we investigate the forces that occur when two objects "interact" with each other. First of all, it should be said that it requires "two" entities to create a force (i.e., *it takes two to tango*).

After many experiments, it was discovered that whenever two bodies interact, the two forces that they exert on each other are always *equal in magnitude and opposite in direction*. This can be written mathematically as:

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Newton's 3$^{\text{rd}}$ Law:
If body A exerts a force on body B (an "action"), then body B exerts a force on body A (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies.

Newton's 3rd law is really a result of *conservation of momentum*, a conservation law we will see in Chapter 8.



Ex. 24: The upward normal force exerted by the floor is 620 N on an elevator passenger who weighs 650 N. What are the reaction forces to these two forces? Is the passenger accelerating? If so, what are the magnitude and direction of the acceleration?

6 Free-Body Diagrams

The *free-body* diagrams are a very useful tool for solving problems involving Newton's 1st and 2nd laws.

1. Draw a picture
2. Identify "the system"
3. Draw the external forces acting "on the system."
4. Judiciously choose a coordinate system. If there is acceleration in the problem, choose one of the axes (x or y) along the direction of acceleration.
5. Resolve all the forces along the x and y axes.
6. Apply Newton's 1st or 2nd law as appropriate.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_x = ma_x \qquad \sum F_y = ma_y$$

7. Solve for the unknown quantities: *acceleration, forces, etc.*

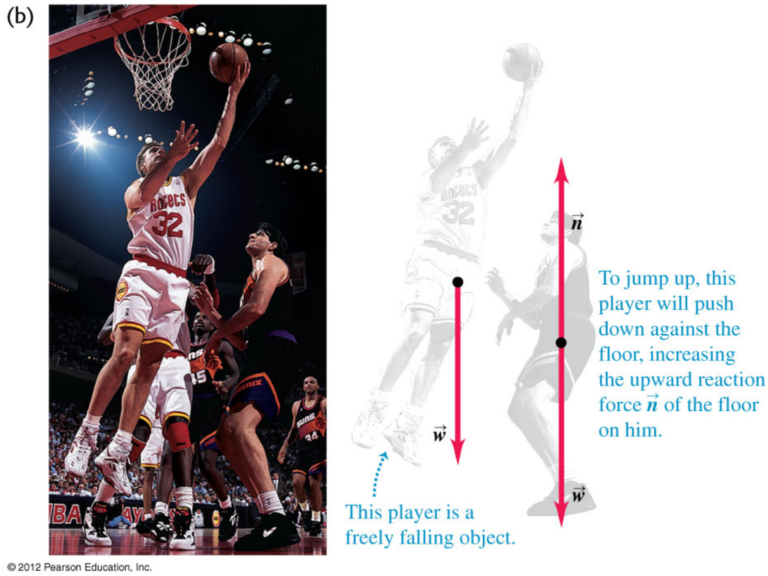


Figure 4: example caption

Ex. 27: Two crates, A and B, sit at rest side-by-side on a frictionless horizontal surface. The crates have masses m_A and m_B. A horizontal force $\vec{F}$ is applied to crate A and the two crates move off to the right. a) Draw clearly labeled free-body diagrams for crate A and for crate B. Indicate which pairs of forces, if any, are third-law action-reaction pairs. b) If the magnitude of force $\vec{F}$ is less than the total weight of the two crates, will it cause the crates to move? Explain.

Prob. 28: A .22 rifle bullet, traveling at 350 m/s, strikes a large tree, which it penetrates to a depth of 0.130 m. The mass of the bullet is 1.80 g. Assume a constant retarding force. a) How much time is required for the bullet to stop? b) What force, in newtons, does the tree exert on the bullet?

Prob. 45: Boxes A and B are connected to each end of a light vertical rope **(Fig. P4.45)**. A constant upward force $F = 80.0$ N is applied to box A. Starting from rest, box B descends 12.0 m in 4.00 s. The tension in the rope connecting the two boxes is 36.0 N. What are the masses of (a) box B, and (b) box A?

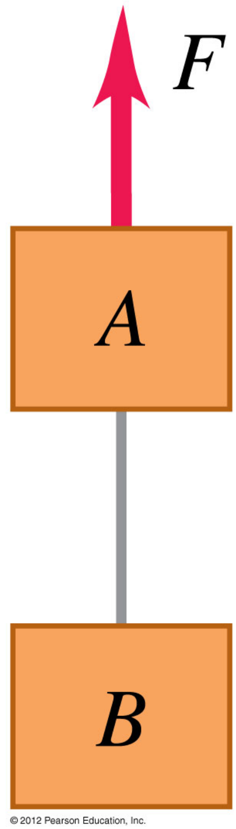


Figure 5: Figure P4.45 from University Physics

End of Chapter