Chapter 1
Units, Physical Quantities, and Vectors

1 The Nature of Physics

- Physics is an experimental science.
- Physicists make observations of physical phenomena.
- Physicists try to find patterns and principles that relate to these phenomena.
- These patterns are called physical theories. And, over time, if they become well established, they are called physical laws or principles.
- The development of physical theories is a two-way process that starts and ends with observations or experiments.
- Physics is the *process* by which we arrive at general principles that describe how the physical universe behaves.
- No theory is every regarded as the final or ultimate truth. New observations may require that a theory be revised or discarded.
- ***It is in the nature of physical theory*** that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.
- Theories typically have a ***range of validity***.

2 Solving Physics problems

- "You don't *know* physics unless you can *do* physics."
- Some guidelines
  1. Identify the relevant concepts
  2. Set up the problem
  3. Execute the solution (i.e., do the math)

4. Evaluate your answer (i.e., Does it make sense? Can you do a consistency check?)

3 Standards and Units

- A **physical quantity** is a number used to describe a physical phenomenon (usually a measurement).
- A physical quantity has units (e.g., mass, length, or time).
- In science, we use the SI (Système International) set of units
  1. Mass – the kilogram
  2. Length – the meter
  3. Time – the second

The kilogram, meter, and second are base units in the SI units. Combinations of these units can be used to describe other physical quantities such as velocity *(m/s)*, and acceleration *(*m/s^2). Sometimes the string of units gets to be so long that we contract them into a new unit called a *derived* unit. For example,

$$\text{A unit of force has base units of} \quad kg \cdot \frac{m}{s^2} \quad \rightarrow \quad \text{newton} \quad \text{or } N$$

where the newton (N) is a derived unit.

3.1 Physical constants

Some physical quantities in nature are "constant" and their values can be found at the NIST website. For example, the speed of light, the mass of the proton, the charge of the electron, and so on.

3.2 Unit Prefixes

Once we've defined a unit, it is easy to introduce larger and smaller units. For example, 1/1000$^{\text{th}}$ of a meter can be abbreviated as 1 mm. A thousand meters can be abbreviated as 1 km. A complete set of prefixes can be found at the NIST web site

4 Unit Consistency and Conversions

An equation must always be dimensionally consistent.

$$\text{distance [meters]} = \text{velocity [meters/second]} \times \text{time [seconds]}$$

Exercise 1.3 How many nanoseconds does it take light to travel 1.00 ft in vacuum? (This result is a useful quantity to remember.)

N.B. For problem 1.9 in the homework, you need to know the *density* of a spherical object. $\rho = \text{mass/volume}$ where the volume $= (4/3)\pi r^3$.

Exercise 1.9 **Neptunium** In the fall of 2002, scientists at Los Alamos National Laboratory determined that the critical mass of Neptunium-237 is about 60 kg. The critical mass of a fissionable material is the minimum amount that must be brought together to start a nuclear chain reaction. Neptunium-237 has a density of 19.5 g/cm^3. What would be the radius of a sphere of this material that has a critical mass?

5 Uncertainty and Significant Figures

5.1 Uncertainties

Whenever a physical quantity is measured, it should be quoted in the following manner:

$$\ell \; = \; (4.56 \pm 0.04)\,\text{cm} \qquad = \quad x \pm \delta x$$

where x is the *mean* or "best" value that can be determined, and δx is the *uncertainty* associated with the measurement.

The **accuracy** of a measured quantity is how close it is to the true value.

The **precision** of a measurement refers to the quality of the measurement. This is usually quoted by calculating the relative uncertainty:

$$\%\ \text{relative uncertainty} \qquad \frac{\delta x}{x} \times 100\% = \frac{0.04}{4.56} \times 100\% = 0.88\%$$

Exercise 1.13 A useful and easy-to-remember approximate value for the number of seconds in a year is $\pi \times 10^7$. Determine the percent error in this approximate value. In other words,"What is the accuracy of this approximation?" (There are 365.24 days in one year.)

5.2 Significant Digits

Upon making a measurement, it is important to record the correct number of significant digits. Furthermore, it is important to maintain the correct number of significant digits in subsequent calculations. The number of significant digits quoted in the final answer is one of the motivating reasons for developing *scientific notation*. Here is a website describing some of the general rules for maintaining the correct number of significant digits.

Exercise 1.12 The volume of a solid cylinder is given by $V = \pi r^2 \cdot h$, where r is the radius and h is the height. You measure the radius and height of a thin cylindrical wire and obtain the results $r = 0.036$ cm and $h = 12.1$ cm. What do your measurements give for the volume of the wire in mm^3? Use the correct number of significant figures in your answer.

6 Estimates and Orders of Magnitude

Once we know the precision of the numbers we are manipulating, we can make educated guesses (or estimates) of our results. Whenever we perform multiple calculations on a calculator, we hope that our final result is *close* to our "best guess." If not, we should re-do our calculation or rethink how we came up with **order-of-magnitude estimate**.

Example: How thick are the pages in your textbook?

7 Vectors and Vector Addition

A **scalar** has magnitude only.

A **vector** has both magnitude and direction.

7.1 Definition of a Vector

The simplest vector $\rightarrow$ the **displacment vector**

Vector notation: $\vec{A}$ (with an arrow over the top) or **A** (boldface)

The **negative** of a vector

Vectors that are **parallel** or **antiparallel**

The magnitude of a vector: (Magnitude of $\vec{A}$) $=$ A or $= |\vec{A}\,|$

7.2 Vector Addition (graphically)

Suppose a *displacement* described by $\vec{C}$ is the result of two displacement vectors, vector $\vec{A}$ followed by vector $\vec{B}$. How can we graphically represent the sum of these two vectors?

$$\vec{C} = \vec{A} + \vec{B}$$

How can we graphically represent the difference between two vectors?

$$\vec{D} = \vec{A} - \vec{B}$$

8 Components of Vectors–numerical addition of vectors

Any vector on the x-y plane can be reduced to the sum of two vectors, one along the x axis, and the other along the y axis.

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

where the magnitude of $\vec{A}_x$ is $A\cos\theta$, and the magnitude of $\vec{A}_y$ is $A\sin\theta$. This is sometimes written as

$$\begin{aligned}\text{magnitude of } \vec{A}_x &= A_x = A\cos\theta \\ \text{magnitude of } \vec{A}_y &= A_y = A\sin\theta\end{aligned}$$

Likewise, if we know A_x and A_y, then we can calculate the magnitude and direction of the vector from the following equations:

$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2} \qquad \text{and} \\ \theta &= \arctan\left(\frac{A_y}{A_x}\right)\end{aligned}$$

Exercise 1.29 For the vectors $\vec{A}$ and $\vec{B}$ in Fig. E1.22, use the method of components to find the magnitude and direction of (a) the vector sum $\vec{A}+\vec{B}$; (b) the vector sum $\vec{B}+\vec{A}$; (c) the vector difference $\vec{A}-\vec{B}$; (d) the vector difference $\vec{B}-\vec{A}$.

Exercise 1.31 A disoriented physics professor drives 3.25 km north, then 2.20 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained using the method of components.

9 Unit Vectors

9.1 Unit vectors in the Cartesian coordinate system

Define the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Now, we can write a vector $\vec{A}$ in terms of the *unit vectors.*

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

10 Product of Vectors

When considering the product of two vectors, there are two kinds of results one can obtain, either a *scalar* or a *vector*.

10.1 Scalar Product

$$\vec{A}\cdot\vec{B} = A\,B\,\cos\phi \qquad \text{(scalar dot product)}$$

Do some examples.

Exercise 1.41 For the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. E1.22, find the scalar products a) $\vec{A}\cdot\vec{B}$; b) $\vec{B}\cdot\vec{C}$; c) $\vec{A}\cdot\vec{C}$.

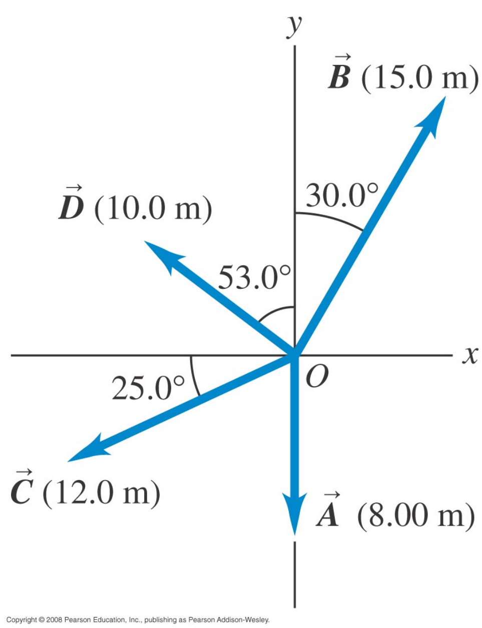

Figure 1: Figure E1.22 from University Physics 15th edition

10.2 Vector Product

$$\vec{C} = \vec{A} \times \vec{B} \qquad \text{(vector cross product)}$$

$$\text{The magnitude of } \vec{C} = A\,B\,\sin\phi$$

Discuss the **right-hand rule** and **right-handed** coordinate systems.

Do some examples.

Exercise 1.45 For the two vectors $\vec{A}$ and $\vec{D}$ in Fig. E1.22 (see the figure above), a) find the magnitude and direction of the vector product $\vec{A} \times \vec{D}$; b) find the magnitude and direction of $\vec{D} \times \vec{A}$.

Exercise 1.xx A cube is placed so that one corner is at the origin and three edges are along the x-, y-, and z-axes of a coordinate system (**Fig. P1.80**). Use vectors to compute (a) the angle between the edge along the z-axis (line ab) and the diagonal from the origin to the opposite corner (line ad), and (b) the angle between the line ac (the diagonal of a face) and line ad.

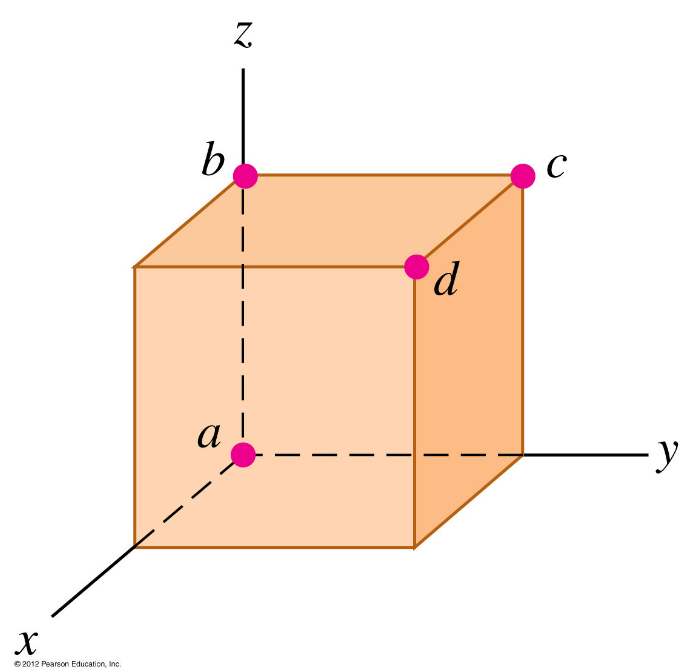

Figure 2: Figure P1.80 from University Physics 14th edition

10.3 Unit Vectors

Let's do an example of a vector cross-product using two 3-dimensional vectors:

$$\vec{A} = 2\hat{\imath} - \hat{\jmath} + \hat{k} \qquad \text{and} \qquad \vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{\imath}(-3-2) + \hat{\jmath}(1-6) + \hat{k}(4+1) \tag{1}$$

$$\vec{C} = \vec{A} \times \vec{B} = \left(-5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}\right) \tag{2}$$

We can also write this vector in a "graphical" representation $\vec{C} = |\vec{C}|\,\hat{C}$ where $\hat{C}$ is a unit vector pointing in the direction of $\vec{C}$. Converting $\vec{C}$ into a *unit vector* is a straight-forward process. Multiply $\vec{C}$ by a constant a such that a **expands** or **shrinks** $\vec{C}$ into a vector of unit length.

$$\hat{C} = a\,\vec{C} = a(-5\hat{\imath} - 5\hat{\jmath} + 5\hat{k}) \tag{3}$$

From the definition of a unit vector, we know that $\hat{C} \cdot \hat{C} = 1$. So,

$$\hat{C} \cdot \hat{C} = a^2(25 + 25 + 25) = 1 \tag{4}$$

Solving for a, and taking the *positive* square-root because we don't want to *flip* the direction of $\hat{C}$, we find that $a = 1/\sqrt{75} = 1/(5\sqrt{3})$. Now we can write $\vec{C}$ in its "graphical" representation as:

$$\vec{C} = |\vec{C}|\,\hat{C} \tag{5}$$

where $|\vec{C}| = \sqrt{25 + 25 + 25} = 5\sqrt{3}$, and $\hat{C} = \frac{1}{\sqrt{3}}(-\hat{\imath} - \hat{\jmath} + \hat{k})$.

You can check that $\hat{C}$ is a *unit vector* by confirming that $\hat{C} \cdot \hat{C} = 1$.

10.4 Questions and Answers on Vector Products

Prob. 1.70 A fence post is 52.0 m from where you are standing, in a direction 37.0° north of east. A second fence post is due south from you. How far are you from the second post if the distance between the two posts is 68.0 m?

11 Challenge Problem

1.91 **Navigating the Big Dipper.** All the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. One light year (ly) is 9.461×10^{15} m.

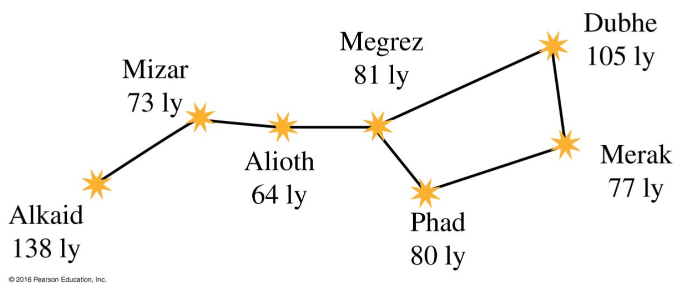

Figure 3: University Physics 15^{th} edition figure P1.91 for problem 1.91

a.) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light years from Alkaid to Merak.

b.) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Extra Problems

Exercise D: Given the following three vectors:
$\vec{A} = 4.0\hat{i} - 8.0\hat{j}$,
$\vec{B} = \hat{i} + \hat{j}$, and
$\vec{C} = -2.0\hat{i} + 4.0\hat{j}$,

(a) Calculate $\vec{A} + \vec{B} + \vec{C} = ?$, and

(b) $\vec{A} - \vec{B} + \vec{C} = ?$