Chapter 9 In Class Problems

Ex. 4

A fan blade rotates with angular velocity $\omega_z(t) = \gamma - \beta t^2$ where $\gamma = 5.00 \ \frac{rad}{s}$ and $\beta = 0.800 \ \frac{rad}{s^3}$

a.) $\alpha(t) = ?$ $\quad \alpha(t) = \frac{d}{dt}\omega_z(t) = -2\beta t = \boxed{-1.60 \ \frac{rad}{s^3} \ t}$

b.) $\alpha(3.00 \ s) = ?$ $\quad \alpha(3.00 \ s) = -1.60 \ \frac{rad}{s^3}(3.00 s) = \boxed{-4.8 \ \frac{rad}{s^2}}$

$$\alpha_{AV} = \frac{\omega(3.00 \ s) - \omega(0.00 \ s)}{(3.00 - 0.00)s} = \frac{\cancel{\gamma} - \beta(3s)^2 - (\cancel{\gamma} - \beta(0)^2)}{3.00 \ s}$$

$$\alpha_{AV} = \frac{-0.800 \ \frac{rad}{s^3}(9 s^2) - 0}{3.00 \ s}$$

$$\boxed{\alpha_{AV}(0 \rightarrow 3 \ s) = -2.4 \ \frac{rad}{s^2}}$$

Ex. 25

An advertisement claims that a centrifuge takes up only 0.127 m of bench space.

$a_{rad} = 3000 g$

$$\omega = 5{,}000 \ \frac{rev}{min}\left(\frac{2\pi \ rad}{rev}\right)\left(\frac{1 \ min}{60 \ s}\right) = 523.6 \ rad/s$$

Conv. Conv.

$$a_{rad} = R\omega^2 \qquad R = \frac{a_{rad}}{\omega^2} = \frac{3{,}000(9.8 \ m/s^2)}{(523.6 \ rad/s)^2} = 0.107 \ m$$

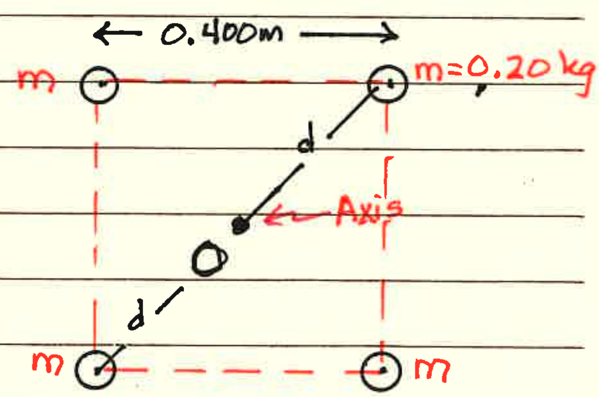
minimum diameter

2R = diameter = 0.214 meters; however, the device only occupies 0.127 m. The claim is NOT realistic

Ex. 30

4 small spheres, ...

a.) Find the moment of inertia with an axis perpendicular to the plane through the point O.



$d = 0.200 m \sqrt{2} = 0.283 \ m$

Chapter 9 In Class Problems

Ex. 30 Cont'd

$$I = md^2 + md^2 + md^2 + md^2$$
$$= m(4d^2) = 4md^2 = 4(0.200\,kg)(0.283\,m)^2$$
$$\boxed{I = 0.064\ kg \cdot m^2}$$

b.) Calculate "I" if the axis is along AB as shown in the figure. Figure E9.30 in my notes.

$I = \sum m_i d_i^2$ where $d_i = 0.200\ m$ for all 4 masses

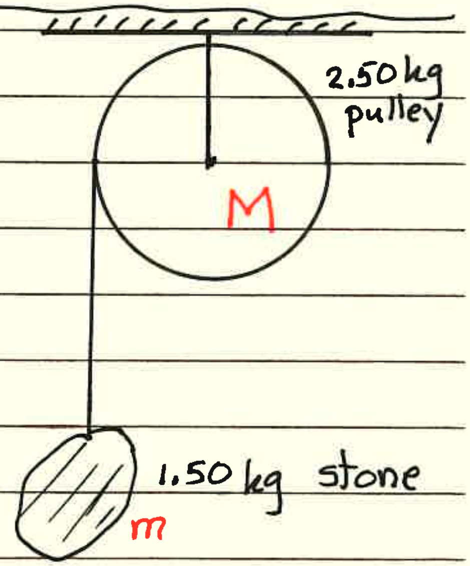
$$I = 4md^2 = 4(0.200\,kg)(0.200\,m)^2 = \boxed{0.032\ kg \cdot m^2}$$

c.) Calculate "I" if the axis passes through the upper-left and lower-right spheres.

$$I = \sum m_i d_i^2 = m(0)^2 + \overset{0.200\,kg}{m}(0.283\,m)^2 + m(0)^2 + \overset{0.200\,kg}{m}(0.283\,m)^2$$

$$\boxed{I = 0.032\ kg \cdot m^2}$$

Ex. 47 A frictionless pulley has the shape of a uniform solid disk of mass 2.50 kg and radius 20.0 cm



a.) How far must the stone fall so that $K_{pulley} = 4.50\ J$

Work-Energy Theorem

$$W_{gr} = K_{pulley} + K_{stone}$$
$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V^2}{R^2} + \frac{1}{2}mv^2$$
$$mgh = \underbrace{\frac{1}{4}Mv^2}_{4.50\ J} + \frac{1}{2}mv^2$$

Chapter 9 In Class Problems

Ex. 47 Cont'd $\frac{1}{4}Mv^2 = 4.50\text{ J}$ $v^2 = \frac{4(4.50\text{ J})}{M = 2.50\text{ kg}} = 7.20\text{ m}^2/\text{s}^2$

$V = 2.683\text{ m/s}$

$$mgh = \frac{1}{2}v^2\left(\frac{1}{2}M + m\right) = \frac{1}{2}(7.20\text{ m}^2/\text{s}^2)(1.25 + 1.50)\text{kg}$$

$$mgh = 9.90\text{ J} \qquad h = \frac{9.90\text{ J}}{mg} = \frac{9.90\text{ J}}{(1.50\text{ kg})9.8\text{ m/s}^2} = 0.673\text{ m}$$

b.) What percent of the total KE does the pulley have?

$$\text{fraction} = \frac{K_{pulley}}{K_{pulley} + K_{stone}} = \frac{\frac{1}{4}Mv^2}{\frac{1}{2}v^2\left(\frac{1}{2}M + m\right)} = \frac{\frac{1}{2}M}{\left(\frac{1}{2}M + m\right)}$$

$$\text{fraction} = \frac{\frac{1}{2}(2.50\text{ kg})}{\frac{1}{2}(2.50\text{ kg}) + (1.50\text{ kg})} = 0.455 \text{ or } 45.5\%$$

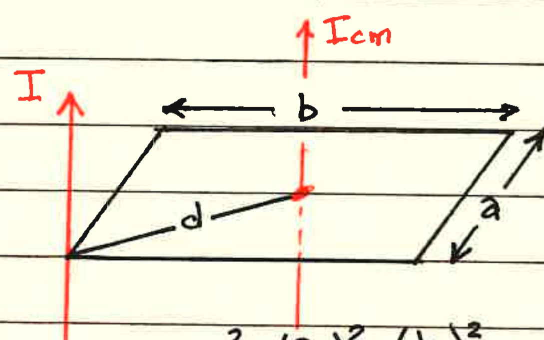
Ex. 53 A thin, rectangular sheet of metal has mass M

$$I = I_{cm} + Md^2$$

$$I = \frac{1}{12}M(a^2 + b^2) + M\left(\frac{a^2}{4} + \frac{b^2}{4}\right)$$

$$I = M\left[a^2\underbrace{\left(\frac{1}{12} + \frac{1}{4}\right)}_{1/3} + b^2\underbrace{\left(\frac{1}{12} + \frac{1}{4}\right)}_{1/3}\right]$$

$$\boxed{I = \frac{1}{3}M(a^2 + b^2)}$$



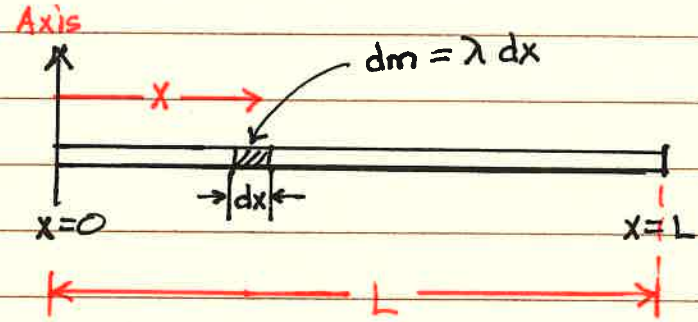
$$d^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$$

Chapter 9 In Class Problems

Ex. 56 Use $I=\int r^2\,dm$ to calculate the moment of inertia of a slender, uniform rod of mass M and length L . . .

$$dI = x^2 dm$$

$$I = \int dI = \int x^2\,dm$$



Where λ = linear mass density

$[\lambda] = kg/m$

$dm = \lambda\,dx \qquad [kg/m][m] = [kg]$

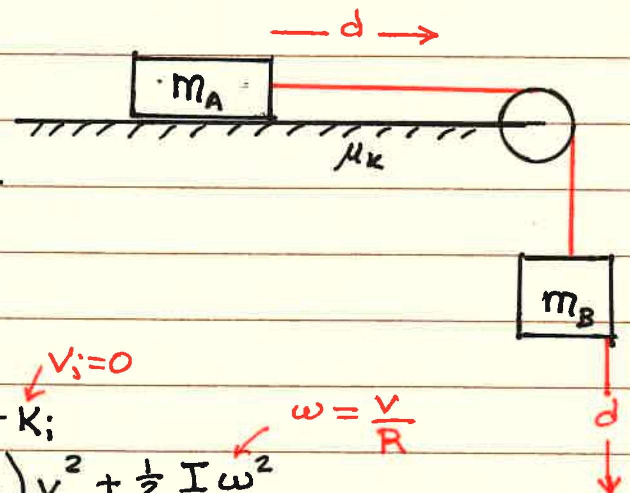
$$I = \int x^2\,dm = \int x^2 \lambda\,dx = \lambda \int_0^L x^2 dx = \frac{\lambda}{3} x^3 \Big|_0^L = \frac{\lambda}{3} L^3$$

Likewise, $M = \int dm = \lambda \int_0^L dx = \lambda x \Big|_0^L = \lambda L$ Therefore, $\lambda = \frac{M}{L}$

$$I = \frac{M/L}{3} L^3 \qquad \boxed{I = \frac{1}{3} M L^2}$$

Same as shown in Fig. 4 in my notes.

Prob. 77 The pulley has a radius R and a moment of Inertia I, and spins on a frictionless axle. Use energy methods to determine $v(d)$.



$$W_{TOT} = \Delta K \Rightarrow W_{gr} + W_{fr} = K_f - K_i \qquad v_i = 0$$

$$\omega = \frac{v}{R}$$

$$m_B g d - \mu_k m_A g d = \tfrac{1}{2}(m_A + m_B) v^2 + \tfrac{1}{2} I \omega^2$$

$$v^2 = \frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2} \qquad \boxed{v = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}}$$