Chapter 11
Equilibrium and Elasticity

1 Conditions for Equilibrium

1st condition for equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \qquad \text{Translational Equilibrium} \tag{1}$$

2nd condition for equilibrium

$$\sum \vec{\tau} = \vec{\mathbf{0}} \qquad \text{Rotational Equilibrium} \tag{2}$$

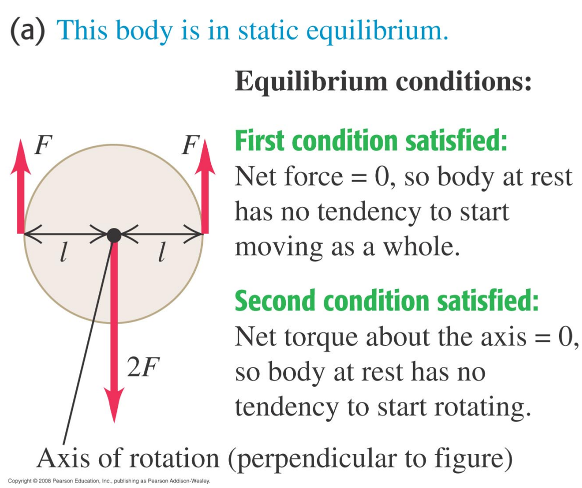


Figure 1: Figure 11.1 from University Physics 12th edition.

2 Center of Gravity

$$x_{\mathrm{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$y_{\mathrm{cm}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$z_{\mathrm{cm}} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$\vec{\mathbf{r}}_{\mathrm{cm}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + m_3\vec{\mathbf{r}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = x_{\mathrm{cm}}\hat{\imath} + y_{\mathrm{cm}}\hat{\jmath} + z_{\mathrm{cm}}\hat{k} \tag{3}$$

$$\vec{\mathbf{r}}_{\mathrm{cm}} = \frac{\sum_i m_i\vec{\mathbf{r}}_i}{m_1 + m_2 + m_3 + \cdots = M}$$

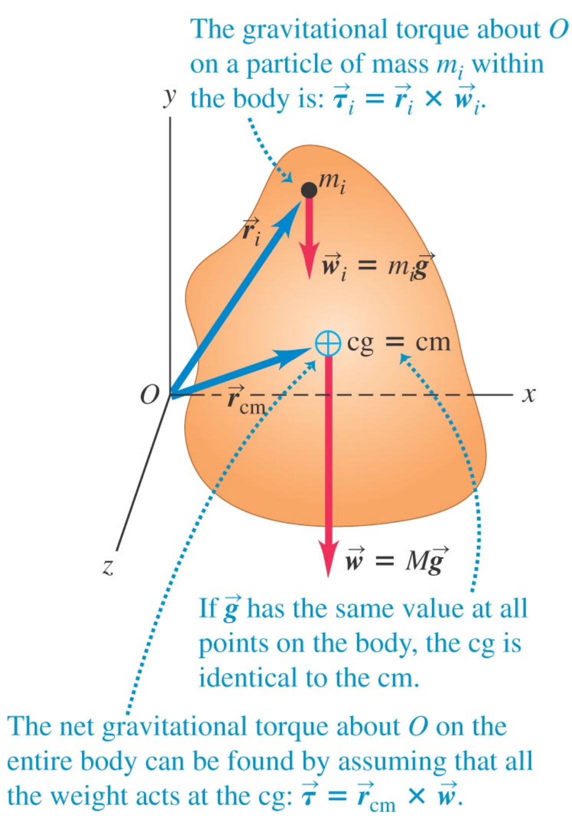


Figure 2: Figure 11.2 from University Physics. Center of gravity (cg).

The sum of the torques under the influence of gravity can be written as:

$$\vec{\tau} = \sum_i \vec{r}_i \times \vec{w}_i = \sum_i \vec{r}_i \times m_i \vec{g} = \left(\sum_i m_i \vec{r}_i \right) \times \vec{g} = \vec{\mathbf{r}}_{\text{cm}} \times M\vec{\mathbf{g}} = \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{w}}$$

What does this mean?

When calculating the torque (τ) for an extended body, and the torque is due to gravity, you can calculate the torque as if the all the mass were concentrated at the object's *center of mass.*

Ex. 3 A uniform rod is 2.00 m long and has a mass 1.80 kg. A 2.40 kg clamp is attached to the rod. How far should the center of gravity of the clamp be from the left-hand end of the rod in order for the center of gravity of the composite object to be 1.20 m from the left-hand end of the rod?

3 Solving Rigid-Body Equilibrium Problems

Ex. 8 Two people are carrying a uniform wooden board that is 3.00 m long and weights 160 N. If one person applies an upward force equal to 60 N at one end, at what point does the other person lift?

Ex. 12 A uniform ladder 5.0 m long rests against a frictionless, vertical wall with its lower end 3.0 m from the wall. The ladder weighs 160 N. The coefficient of static friction between the foot of the ladder and the ground is 0.40. A man weighing 740 N climbs slowly up the ladder. a) What is the maximum frictional force that the ground can exert on the ladder at its lower end? b) What is the actual frictional force when the man has climbed 1.0 m along the ladder? c) How far along the ladder can the man climb before the ladder starts to slip?

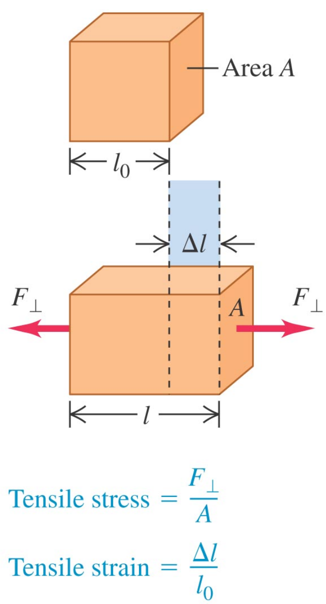
4 Stress, Strain, and Elastic Moduli

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic Modulus} \qquad \text{(Hooke's Law)}$$

4.1 Tensile and Compressive Stress and Strain

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp}{A}\frac{\ell_o}{\Delta\ell}$$

Introduce the units of pressure (F/A), the **pascal**.



$$1\,\text{Pa} = 1\,\text{N/m}^2 \qquad 1\,\text{psi} = 6895\,\text{Pa}$$

Examples of the Young's Modulus. Table 11.1

Ex. 27 A circular steel wire 2.00 m long must stretch no more than 0.25 cm when a tensile force of 700 N is applied to each end of the wire. What minimum diameter is required for the wire?

Table 11.1 Approximate Elastic Moduli

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Crown glass	6.0×10^{10}	5.0×10^{10}	2.5×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

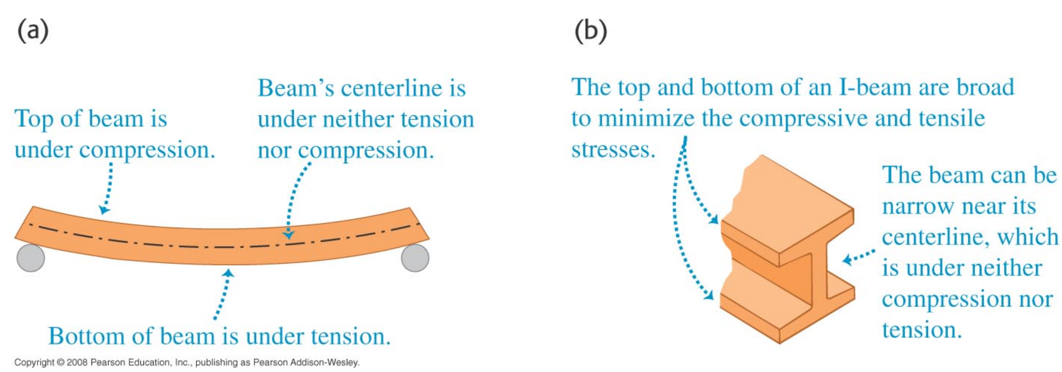


Figure 3: Figure 11.16 from University Physics

4.2 Bulk Stress and Strain

Definition of pressure: $p = F_\perp / A$

$$\text{Bulk Modulus} = B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V / V_o} \qquad \text{(Bulk modulus)}$$

The bulk modulus B has the same units as pressure, namely Pa.

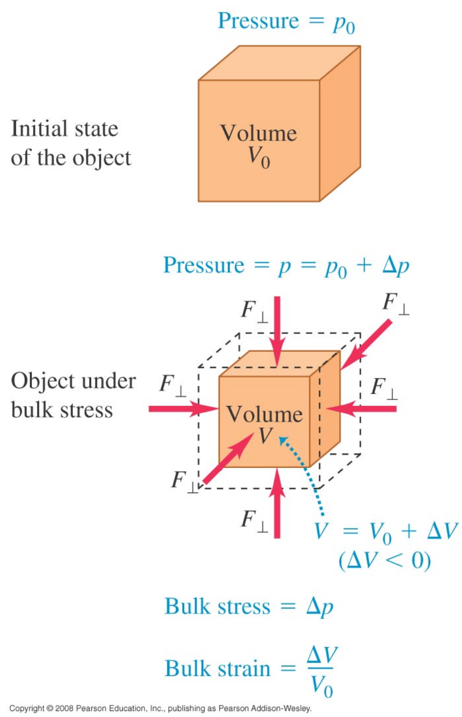


Figure 4: Figure 11.17 from University Physics

The reciprocal of the bulk modulus is called the **compressibility**, k.

$$k = \frac{1}{B} = -\frac{1}{V_o}\frac{\Delta V}{\Delta p}$$

Higher values of k, means that the material is easier to compress.

$$k_{\text{water}} = 45.8 \times 10^{-11}\,\text{Pa}^{-1}$$

Ex. 36 In the Challenger Deep of the Marianas Trench, the depth of seawater is 10.9 km and the pressure is 1.16×10^8 Pa (about 1.15×10^3 atm). a) If a cubic meter of water is taken from the surface to this depth, what is the change in its volume? (Normal atmospheric pressure is about 1.0×10^5 Pa. Assume that k for seawater is the same as the freshwater value given in Table 11.2.) b) What is the density of seawater at this depth? (At the surface, seawater has density of 1.03×10^3 kg/m^3. $k_{\text{water}} = 45.8 \times 10^{-11}$ Pa^{-1}

Table 11.2 Compressibilities of Liquids

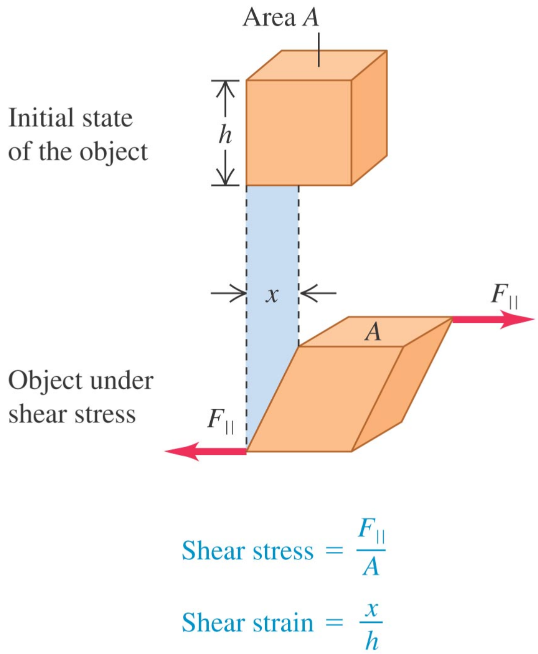
Liquid	Compressibility, k Pa^{-1}	Compressibility, k atm^{-1}
Carbon disulfide	93×10^{-11}	94×10^{-6}
Ethyl alcohol	110×10^{-11}	111×10^{-6}
Glycerine	21×10^{-11}	21×10^{-6}
Mercury	3.7×10^{-11}	3.8×10^{-6}
Water	45.8×10^{-11}	46.4×10^{-6}

4.3 Shear Stress and Strain

$$\text{Shear stress} = \frac{F_\parallel}{A}$$

$$\text{Shear strain} = \frac{x}{h}$$

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_\parallel/A}{x/h} = \frac{F_\parallel}{A}\frac{h}{x} \qquad \text{(Shear modulus)}$$



Prob. 74 You are trying to raise a bicycle wheel of mass m and radius R up over a curb of height h. To do this, you apply a horizontal force $\vec{F}$ **(Fig. P11.74)**. What is the smallest magnitude of the force $\vec{F}$ that will succeed in raising the wheel onto the curb when the force is applied (a) at the center of the wheel and (b) at the top of the wheel? (c) In which case is less force required?