Chapter 13 Solutions In-Class

Ex. 1

What is the ratio of the gravitational pull of the sun on the moon to that of the earth on the moon?

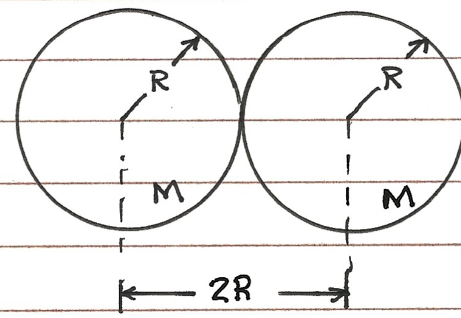
$$\frac{F_{sun/moon}}{F_{earth/moon}} = \frac{GM_s m / d_{sun}^2}{GM_E m / d_{earth}^2} = \frac{M_s}{M_e}\left(\frac{d_{earth}}{d_{sun}}\right)^2$$

$$\frac{F_{sun/moon}}{F_{earth/moon}} = \frac{1.99\times10^{30}\ kg}{5.97\times10^{24}\ kg}\left(\frac{3.84\times10^{8}\ m}{1.50\times10^{11}\ m}\right)^2 = 2.18$$

Ex. 4

Two uniform spheres, each with mass M and radius R, touch one another.

$$F = \frac{G\,MM}{(2R)^2} = \frac{GM^2}{4R^2}$$



Ex. 14

Rhea, one of Saturn's moons, has a radius of 765 km and an acceleration due to gravity of 0.278 m/s^2 at its surface.

$M = ?$ $\rho = ?$

$$a = \frac{GM}{R^2} \qquad M = \frac{aR^2}{G} = \frac{0.278\ m/s^2\ (765\times10^3\ m)^2}{6.67\times10^{-11}\ N\cdot m^2/kg^2}$$

$$\boxed{M = 2.44\times10^{21}\ kg}$$

$$\rho = \text{density} = \frac{\text{mass}}{\text{volume}} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2.44\times10^{21}\ kg}{4/3\,\pi\,(765\times10^3\ m)^3}$$

$$\boxed{\rho = 1.30\times10^3\ kg/m^3}$$

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Ex. 10

A planet orbiting a distant star has a radius 3.24×10^6 m.

$V_{esc} = 7.65 \times 10^3$ m/s $= \sqrt{\frac{GM\,2}{R}}$ $\qquad V_{esc}^2 = 2R\left(\frac{GM}{R^2}\right)$

g = acceleration due to gravity $= \frac{GM}{R^2}$. So, $V_{esc}^2 = 2Rg$

$$g = \frac{V_{esc}^2}{2R} = \frac{(7.65 \times 10^3 \text{ m/s})^2}{2(3.24 \times 10^6 \text{ m})} = \boxed{9.03 \text{ m/s}^2}$$

Ex. 25

For a satellite to be in a circular orbit 890 km above the surface of the earth, ...

a.) $V_{orb} = ?$ $\qquad V_{orb} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_E}{R_E + h}} = \sqrt{\frac{6.67 \times 10^{-11}\ 5.98 \times 10^{24}}{6.37 \times 10^6 + 890 \times 10^3}}$

$$\boxed{V_{orb} = 7412 \text{ m/s}}$$

b.) $T = ?$ $\qquad 2\pi r = V_{orb} T \qquad T = \frac{2\pi(R_E + h)}{V_{orb}} = \frac{2\pi(6.37 \times 10^6 + 890 \times 10^3)}{7412 \text{ m/s}}$

$$T = 6154 \text{ s}\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = \boxed{1.71 \text{ hours}}$$

Ex. 30

Hot Jupiters. In 2004 astronomers reported the discovery of a large Jupiter-sized planet

$r = (1/9)\, r_{Mercury} = \frac{1}{9}(5.79 \times 10^{10} \text{ m}) = 6.43 \times 10^9$ m

$T = 3.09 \text{ days}\left(\frac{86{,}400 \text{ s}}{1 \text{ day}}\right) = 2.67 \times 10^5$ sec. $\qquad M = M_{star} = ?$

$$T^2 = \frac{4\pi^2 r^3}{GM} \qquad M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (6.43 \times 10^9)^3}{(6.67 \times 10^{-11})(2.67 \times 10^5)^2}$$

$$\boxed{M = 2.21 \times 10^{30} \text{ kg}}$$

b.) $v = ?$ $\qquad V = \frac{2\pi r}{T} = \frac{2\pi(6.43 \times 10^9)}{2.67 \times 10^5}$ $\qquad \boxed{v = 151.4 \text{ km/s}}$

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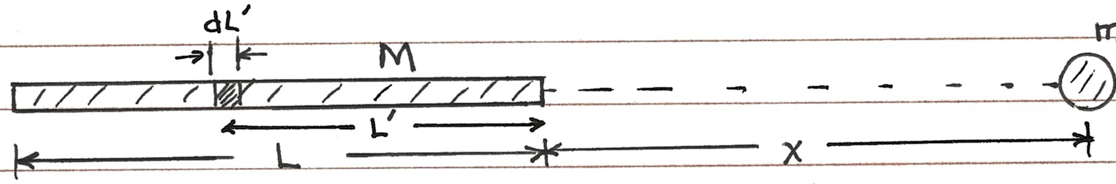
Ex. 38 A thin, uniform rod has length L and mass M. A small uniform sphere of mass m is placed a distance x from one end of the rod .... $U_{gr} = ?$



The solution for this problem can be found in my lecture notes.

$$dU = -\frac{G m \lambda \, dL'}{X + L'}$$

$$U_{gr} = -G m \lambda \int_0^L \frac{dL'}{X + L'}$$

$$U_{gr} = -G m \lambda \ln\left(\frac{L + X}{X}\right) = -G m \lambda \ln\left(1 + \frac{L}{X}\right)$$

From Appendix C $\Rightarrow \ln(1 + z) = z - \frac{z^2}{2} + \ldots$

$$U_{gr} = -G m \lambda \left(\frac{L}{X} - \frac{L^2}{2X^2} + \ldots\right) = -\frac{G m (\underbrace{\lambda L}_{M})}{X}\left(1 - \frac{L}{2X} + \ldots\right)$$

$$U_{gr} \cong -\frac{G M m}{X}\left(1 - \frac{L}{2X}\right) \qquad \text{For } X \gg L \quad U_{gr} \to -\frac{G M m}{X}$$

$$F_{gr} = -\frac{dU}{dX} = -\frac{d}{dX}\left(-G m \lambda \ln\left(1 + \frac{L}{X}\right)\right) = +G m \lambda \frac{\left(-L/X^2\right)}{1 + L/X}$$

$$\boxed{F_{gr} = -\frac{G m \overbrace{(\lambda L)}^{M}}{X^2 + LX}} \qquad F_{gr}(X) = -\frac{G M m}{X^2} \frac{1}{1 + L/X}$$

As $X \gg L$, then $\frac{1}{1 + L/X} \to 1$, then $\boxed{F_{gr}(X) \cong -\frac{G M m}{X^2}}$

... as expected for a point-like object.

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Ex. 43 At the Galaxy's Core. Astronomers have observed a small massive object at the center of our Milky Way galaxy.

Diameter = 15 light-years $= 15(3\times10^{8}\,m/s)(3.16\times10^{7}\,s) = 1.42\times10^{17}\,m$

velocity = 200 km/s $= 200\times10^{3}\,m/s$

$M = ?$

V = velocity of a circular orbit $= \sqrt{\dfrac{GM}{r}}$ $\qquad v^2 = \dfrac{GM}{r}$

$$M = \frac{rv^2}{G} = \frac{(7.10\times10^{6}\,m)(200\times10^{3}\,m/s)^2}{6.67\times10^{-11}\,N\cdot m^2/kg^2} = 4.26\times10^{37}\,kg$$

$$M = 4.26\times10^{37}\,kg\left(\frac{1\,M_\odot\ (\text{solar mass})}{1.99\times10^{30}\,kg}\right) = \boxed{2.14\times10^{7}\,M_\odot}$$

b.) No, it cannot be a single ordinary star. It's mass is HUGE.

c.) R_s = Schwarzschild radius $= \dfrac{2GM}{c^2} = \dfrac{2(6.67\times10^{-11})\,4.26\times10^{37}}{(3.00\times10^{8})^2}$

$\boxed{R_s = 6.31\times10^{10}\,m}$ Yes. It will fit inside the earth's orbit around the sun. $1.50\times10^{11}\,m$

Prob. 56 Your starship, the Aimless Wanderer, lands on the mysterious planet Mongo. A 2.00 kg stone $V_{oy} = 12.0\,m/s$ $t = 4.80\,s$

$y = 0$ $\qquad M = ?$

Eq. 3 ⇒ $\overset{=0}{y} = V_{oy}t - \frac{1}{2} g_{Mongo} t^2$ $\qquad g_{Mongo} = \dfrac{2V_{oy}}{t} = \dfrac{2(12.0\,m/s)}{4.80\,s}$

$g_{Mongo} = 5.00\,m/s^2$ $\qquad$ a.) $M = ?$ $\qquad g_{Mongo} = \dfrac{GM_{Mongo}}{R^2_{Mongo}} = \dfrac{GM_{Mongo}}{(C_{Mongo}/2\pi)^2}$ Circumf.

$$M_{Mongo} = \frac{C^2_{Mongo}\,g_{Mongo}}{(2\pi)^2 G} = \frac{(2.00\times10^{8}\,m)^2(5.00\,m/s^2)}{4\pi^2(6.67\times10^{-11}\,N\cdot m^2/kg^2)} = \boxed{7.60\times10^{25}\,kg}$$

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Prob. 56 cont'd

b.) Aimless Wanderer goes into circular orbit 30,000 km above the surface, calculate the period of its orbit.

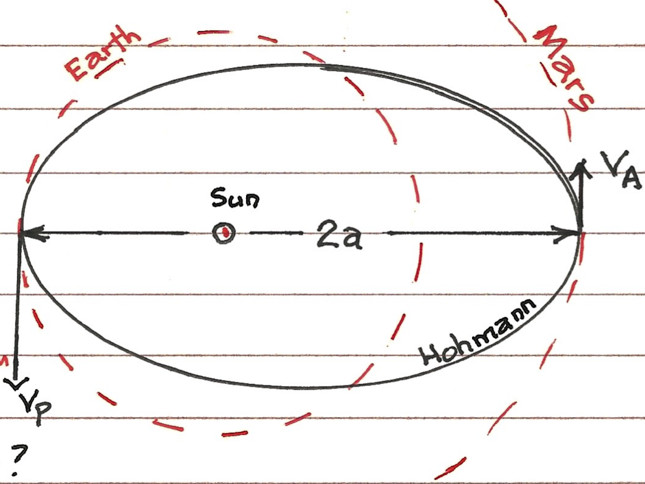
$$R_{Mongo} = \text{Circ.}/2\pi = 2\times10^{8}\,\text{m}/2\pi = 3.18\times10^{7}\,\text{m}$$

$$T = \frac{2\pi}{\sqrt{GM_{Mongo}}}\, r^{3/2} = \frac{2\pi}{\sqrt{GM_{Mongo}}}\left(R_M + h\right)^{3/2}$$

$$T = \frac{2\pi\left(3.18\times10^{7}\,\text{m} + 3.00\times10^{7}\,\text{m}\right)^{3/2}}{\sqrt{(6.67\times10^{-11})(7.60\times10^{25})}} \qquad T = 42{,}874\,\text{s}\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)$$

$$\boxed{T = 11.9 \text{ hours}}$$

Prob. 79 Interplanetary Navigation: The most efficient way to send a spacecraft from the earth to another planet is by using a Hohmann Transfer Orbit.



Earth

$$V_E = \sqrt{\frac{GM_\odot}{d_E}} = 2.98\times10^{4}\,\text{m/s}$$

Mars

$$V_M = \sqrt{\frac{GM_\odot}{d_M}} = 2.41\times10^{4}\,\text{m/s}$$

Aphelion Perihelion

What are the values for V_A and V_P for the elliptical Hohmann transfer orbit?

ellipse Sun satellite = spacecraft

$$E = \frac{-GM_\odot m}{2a} = \frac{-GM_\odot m}{d_E + d_M} = KE(d_E) + PE(d_E)$$

$$\frac{-GM_\odot \cancel{m}}{d_E + d_M} = \frac{1}{2}\cancel{m}V_P^2 - \frac{GM_\odot \cancel{m}}{d_E} \qquad V_P^2 = 2GM_\odot\left(\frac{1}{d_E} - \frac{1}{d_E + d_M}\right)$$

Chapter 13 In-Class Solutions

Prob. 79 cont'd

$$V_P = \sqrt{2(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{1}{1.50\times10^{11}} - \frac{1}{(1.50+2.28)\times10^{11}}\right)}$$

$$\boxed{V_P = 3.27\times10^{4}\ \text{m/s}} \quad > \quad \overset{\text{Earth}}{V_E} = 2.98\times10^{4}\ \text{m/s}$$

$V_A = ?$

$$E_{\text{ellipse}} = -\frac{G\overset{\text{Sun}}{M_\odot}\,\overset{\text{Satellite = spacecraft}}{\cancel{m}}}{d_E + d_M} = \frac{1}{2}\cancel{m}V_A^2 - \frac{GM_\odot \cancel{m}}{d_M}$$

$$V_A^2 = 2GM_\odot\left(\frac{1}{d_M} - \frac{1}{d_M + d_E}\right)$$

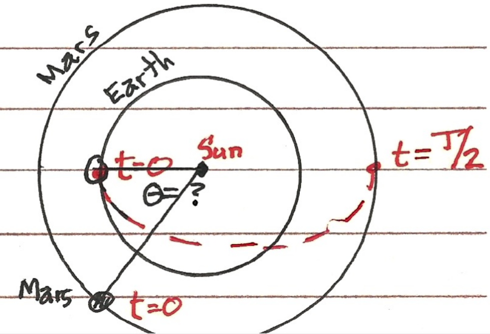
$$V_A = \sqrt{2(6.67\times10^{-11})(1.99\times10^{30})\left(\frac{1}{2.28\times10^{11}} - \frac{1}{(1.50+2.28)\times10^{11}}\right)}$$

$$\boxed{V_A = 2.15\times10^{4}\ \text{m/s}} \quad < \quad \overset{\text{Mars}}{V_M} = 2.41\times10^{4}\ \text{m/s}$$

Earth to Mars: You must increase the speed of the spacecraft in the direction of the earth's orbit (motion)

Mars to Earth: You must decrease the speed of the spacecraft with respect to Mars' orbital vebcity.

$$T = \frac{2\pi}{\sqrt{GM_\odot}}\,a^{3/2}$$ is the period of the full elliptical orbit



However, we only want to know $T/2$, half the period.

The time-of-flight from Earth → Mars.

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Prob. 79 cont'd

$$\frac{T}{2} = \frac{\pi}{\sqrt{GM_\odot}} a^{3/2} \quad \text{where } a = \frac{d_E + d_M}{2} = 1.89 \times 10^{11}\text{ m}$$

$$\frac{T}{2} = \frac{\pi (1.89 \times 10^{11})^{3/2}}{\sqrt{(6.67\times10^{-11})(1.99\times10^{30})}} = 2.24 \times 10^{7}\text{ s} = \boxed{259 \text{ days}}$$

c.) Where should mars be with respect to the earth when launching from earth → mars?

$$T_{Mars} = \text{period for mars' orbit} = \frac{2\pi}{\sqrt{GM_\odot}} a^{3/2} \quad \text{where } a = d_M$$

$$T_{Mars} = \frac{2\pi (2.28 \times 10^{11})^{3/2}}{\sqrt{(6.67\times10^{-11})(1.99\times10^{30})}} = 5.94 \times 10^{7}\text{ s} = \underline{\underline{688 \text{ days}}}$$

Hohmann transfer

$$\text{fraction of mars' orbit} = \frac{T/2}{T_{mars}} = \frac{259}{688} = 0.376$$

$$0.376\,(360^\circ) = 136^\circ \quad \text{in 259 days}$$

See the figure on the previous page for θ

$$\boxed{\theta = 180^\circ - 136^\circ = 44^\circ}$$

Mars must be ahead of the earth by 44° at launch time if it is going to be at the aphelion of the Hohmann transfer orbit.