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DATE

Problem 11.66

Part (b.)

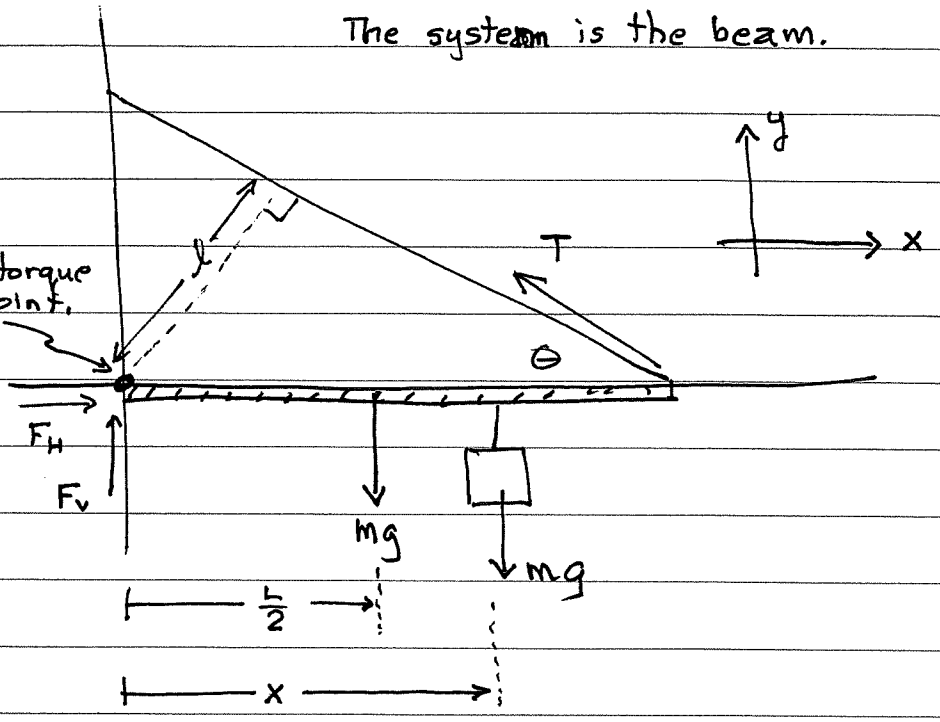
The system is the beam.

① $\Sigma F_x = 0$

② $\Sigma F_y = 0$

③ $\Sigma \tau_z = 0$

Calculate the torque about this point.



5 forces acting on the beam:

F_H, F_V, mg, mg, & T

① $\Sigma F_x = 0$ $\quad F_H - T\cos\theta = 0$

② $\Sigma F_y = 0$ $\quad F_V + T\sin\theta - 2mg = 0$

$$F_V^{max} = \mu_s N = \mu_s F_H$$

③ $\Sigma \tau_z = 0 \Rightarrow \Sigma \tau_z = +Tl - mg\frac{L}{2} - mgx = 0$

$l = L\sin\theta$

③ $T L\sin\theta - mg\frac{L}{2} - mgx = 0$

$$T = \frac{mg\left(\frac{L}{2} + x\right)}{L\sin\theta}$$

$$\boxed{T = \frac{mg\left(\frac{L}{2} + x\right)}{L\sin\theta}} \quad \text{Eq. (1).}$$

Substitute Eq. (1) into ① and ②

① $F_H - \frac{mg\left(\frac{L}{2} + x\right)}{L\tan\theta} = 0$

$$\boxed{F_H = \frac{mg\left(\frac{L}{2} + x\right)}{L\tan\theta}} \quad \text{Eq. (2)}$$

② $F_V \to F_V^{max} = \mu_s F_H \Rightarrow \mu_s F_H = 2mg - \frac{mg\left(\frac{L}{2} + x\right)}{L\sin\theta}\sin\theta$

$$\boxed{F_H = \frac{mg}{\mu_s}\left(2 - \frac{\left(\frac{L}{2} + x\right)}{L}\right)} \quad \text{Eq. (3)}$$

Set Eq. (2) equal to Eq. (3) and solve for "x."

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Problem 11.66 cont'd

Part (b.) cont'd

$$\frac{mg\left(\frac{L}{2}+x\right)}{L\tan\theta} = \frac{mg}{\mu_s}\left(2-\frac{\left(\frac{L}{2}+x\right)}{L}\right)$$

$$\frac{\left(\frac{L}{2}+x\right)}{\tan\theta} = \frac{1}{\mu_s}\left(2L-\left(\frac{L}{2}+x\right)\right)$$

$$\mu_s\left(\frac{L}{2}+x\right) = \tan\theta\left(\frac{3L}{2}-x\right)$$

$$\boxed{\mu_s\frac{L}{2}+\mu_s x = \tan\theta\,\frac{3L}{2} - x\tan\theta} \quad \text{Eq. (4)}$$

$$x\left(\mu_s+\tan\theta\right) = \frac{3L}{2}\tan\theta - \mu_s\frac{L}{2} = \frac{L}{2}\left(3\tan\theta-\mu_s\right)$$

$L = 100\text{ cm}$

$\theta = 18^\circ$

$\mu_s = 0.39$

$$x = \left(\frac{L}{2}\right)\frac{\left(3\tan\theta-\mu_s\right)}{\left(\mu_s+\tan\theta\right)} = 50\text{cm}\,\frac{\left(3\tan(18^\circ)-0.39\right)}{\left(0.39+\tan(18^\circ)\right)}$$

$$\boxed{x = 40.9\text{ cm}}$$

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c.) Solve for μ_s $\Rightarrow$ start with Eq (4)

$$\mu_s\left(\frac{L}{2}+x\right) = \tan\theta\left(\frac{3L}{2}-x\right)$$

$$\mu_s = \tan\theta\,\frac{\left(\frac{3L}{2}-x\right)}{\left(\frac{L}{2}+x\right)} \qquad \boxed{\mu_s = \tan\theta\,\frac{\left(3L-2x\right)}{L+2x}}$$

$\theta = 18^\circ$ $x = 15\text{ cm}$ $L = 100\text{ cm}$

$$\mu_s = \tan(18^\circ)\frac{\left(300-30\right)}{100+30} = \tan(18^\circ)\left(\frac{270}{130}\right)$$

$$\mu_s = 0.6748 \qquad \boxed{\mu_s = 0.67}$$